The Skellam distribution

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Consider

$$X \sim \text{Poisson}(\lambda_1)$$

and

$$Y \sim \text{Poisson}(\lambda_2)$$

then

$$Z = X - Y$$

has a Skellam distribution with

$$Z \sim \text{Skellam}(\lambda_2, \lambda_2).$$

See Wikipedia. *Skellam distribution* `http://en.wikipedia.org/wiki/Skellam_distribution`

Load the package

```
library('skellam')
```

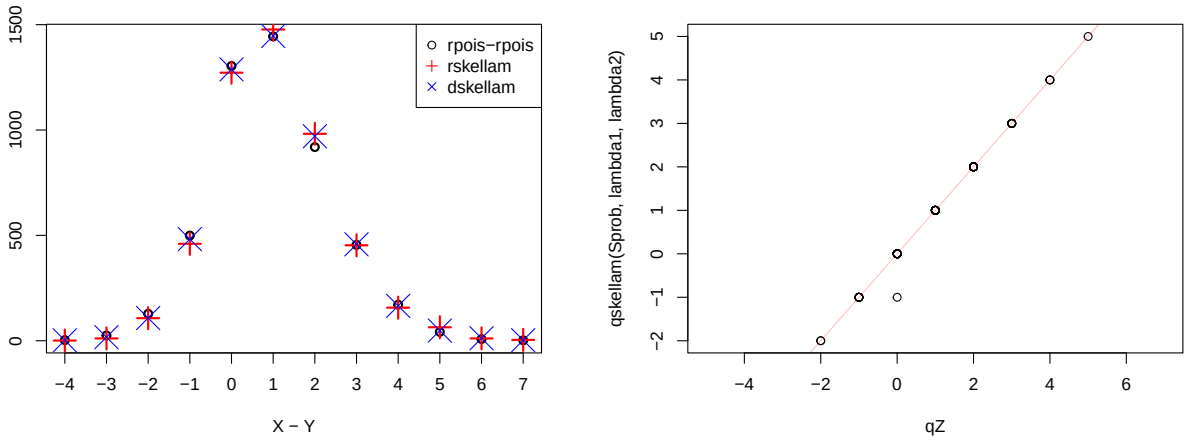
Set some parameters

```
N = 5000
lambda1 = 1.5
lambda2 = 0.5
```

Simulate Poisson and Skellam random variables

```
X = rpois(N, lambda1)
Y = rpois(N, lambda2)
XminusY = X - Y
Z = rskellam(N, lambda1, lambda2)
```

Produce figures

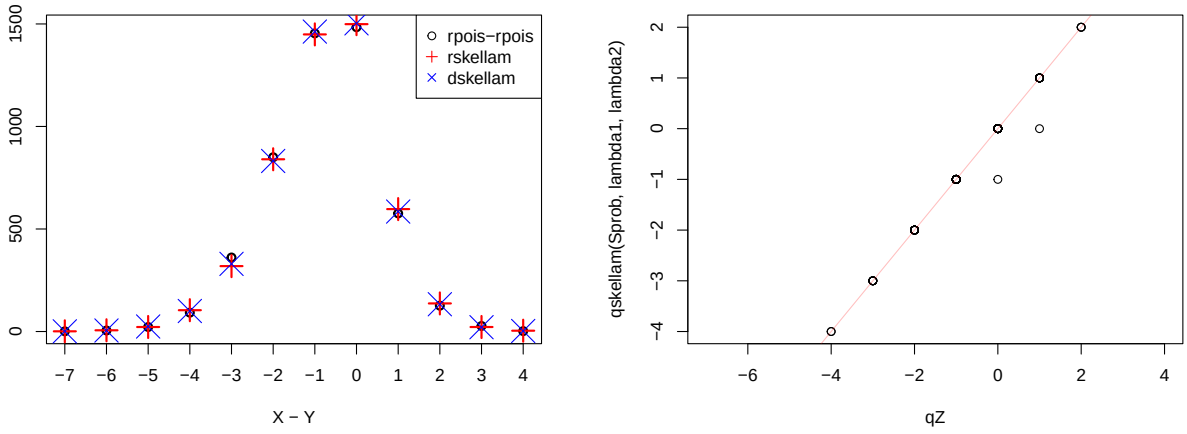


(a) densities (b) quantiles

Figure 1: Differences of Poisson and Skellam with parameters 1.5 and 0.5

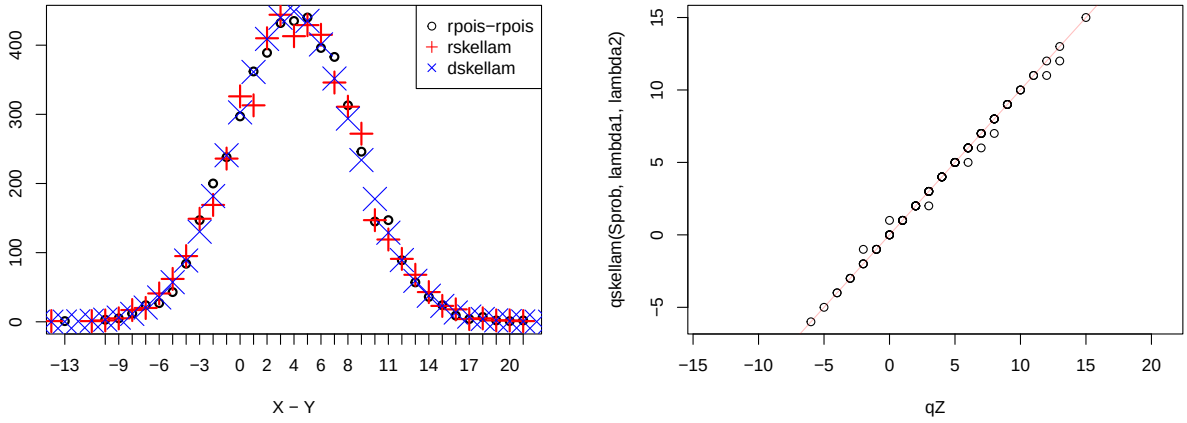
```
plot(table(XminusY), xlab='X - Y', ylab='', type='p', pch=1)
points(table(Z), col='red', type='p', pch=3, cex=2)
xseq = seq(floor(par('usr')[1]), ceiling(par('usr')[2]))
points(xseq, N*dskellam(xseq, lambda1, lambda2), col='blue',
    pch=4, cex=3)
legend('topright', pch=c(1,3,4), col=c('black','red','blue'),
    legend=c('rpois-rpois', 'rskellam', 'dskellam'))

Sprob = seq(0,1,by=1/100)
qZ = quantile(Z, prob=Sprob)
plot(qZ, qskellam(Sprob, lambda1, lambda2))
abline(c(0,1), col='#FF000040')
```

If the `dskellam` and `rskellam` functions are correct, the three sets points on the left will be coincident. If the `qskellam` funciton is correct the points on the right will lie on the red line.



(a) densities (b) quantiles

Figure 2: Differences of Poisson and Skellam with parameters 0.5 and 1.25



(a) densities (b) quantiles

Figure 3: Differences of Poisson and Skellam with parameters 12 and 8