

Finite mixture models in secr

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Variation in detection probability among individuals ('individual heterogeneity') is a persistent problem in capture-recapture studies. Ideally, such variation is removed by grouping individuals into homogeneous classes (males and females) or including continuous predictors such as body weight. Finite mixture models are an option when unmodelled heterogeneity remains (Pledger 2000; Borchers and Efford 2008). The population is assumed to comprise 2 or more latent classes differing in detection parameters, with an unknown proportion in each class. The likelihood is a weighted sum over the classes.

Implementation in secr

Version 1.3 of **secr** introduced the 2-class finite mixture model as an option for any 'real' detection parameter (e.g., g_0 or sigma of a halfnormal detection function). Consider a simple example, using conditional likelihood and `trace = FALSE` for brevity:

```
> library(secr)
> model.0 <- secr.fit(captdata, model = g0~1, CL = TRUE, trace = FALSE)
```

Specify a 2-class mixture by adding the predictor `h2` to the model formula:

```
> model.h2 <- secr.fit(captdata, model = g0~h2, CL = TRUE, trace = FALSE)
> model.h2
```

```
secr.fit( capthist = captdata, model = g0 ~ h2, CL = TRUE,
  trace = FALSE )
secr 2.3.2, 06:19:55 15 May 2012
```

```
Detector type      single
Detector number    100
Average spacing    30 m
```

x-range 365 635 m
y-range 365 635 m
N animals : 76
N detections : 235
N occasions : 5
Mask area : 22.09 ha

Model : $g0^{h2} \sigma^1 \text{pmix}^{h2}$
Fixed (real) : none
Detection fn : halfnormal
N parameters : 4
Log likelihood : -755.6554
AIC : 1519.311
AICc : 1519.874

Beta parameters (coefficients)

	beta	SE.beta	lcl	ucl
g0	-0.8107986	0.53658072	-1.862478	0.2408803
g0.h22	-0.8195996	1.34237095	-3.450598	1.8113991
sigma	3.3808871	0.04494941	3.292788	3.4689863
pmix.h22	-1.2823682	5.11111485	-11.299969	8.7352329

Variance-covariance matrix of beta parameters

	g0	g0.h22	sigma	pmix.h22
g0	0.287918870	0.538223652	-0.001457309	2.58386006
g0.h22	0.538223652	1.801959764	0.004832821	6.08285864
sigma	-0.001457309	0.004832821	0.002020449	0.01914592
pmix.h22	2.583860057	6.082858643	0.019145920	26.12349506

Fitted (real) parameters evaluated at base levels of covariates

session = 1, h2 = 1

	link	estimate	SE.estimate	lcl	ucl
g0	logit	0.3077203	0.114307	0.1344145	0.5599306
sigma	log	29.3968362	1.322038	26.9178015	32.1041813
pmix	logit	0.7828526	NA	NA	NA

session = 1, h2 = 2

	link	estimate	SE.estimate	lcl	ucl
g0	logit	0.1637758	0.0734865	0.06403927	0.3592273
sigma	log	29.3968362	1.3220382	26.91780146	32.1041813

```
pmix logit 0.2171474          NA          NA          NA
```

From the output you can see that `secr.fit` has expanded the model to include an extra ‘real’ parameter, `pmix` for the proportions in the respective latent classes. You could specify this yourself as part of the `model` argument, but `secr.fit` knows to add it. There are also two extra ‘beta’ parameters: `g0.h22` which is the difference in `g0` between the classes on the link (logit) scale, and `pmix.h22` which is the proportion in the second class, also on the logit scale. Fitted (real) parameter values are reported separately for each mixture class (`h2 = 1` and `h2 = 2`).

We can compare a 2-class finite mixture model to the null (constant) model using AIC:

```
> AIC (model.0, model.h2)

              model  detectfn npar   logLik
model.0          g0~1 sigma~1 halfnormal    2 -755.9344
model.h2 g0~h2 sigma~1 pmix~h2 halfnormal    4 -755.6554
              AIC    AICc dAICc  AICwt
model.0 1515.869 1516.033 0.000 0.8722
model.h2 1519.311 1519.874 3.841 0.1278
```

In this case there is no reason to prefer the mixture model.

More complex models are allowed. For example, one might, somewhat outlandishly, fit a learned response to capture that differs between two latent classes, while also allowing sigma to differ between classes:

```
> model.h2xbh2s <- secr.fit(captdata, model = list(g0~h2*b, sigma~h2),
  CL = FALSE)
```

Number of classes

The theory of finite mixture models in capture–recapture (Pledger 2000) allows an indefinite number of classes – 2, 3 or perhaps more. Programmatically, the extension to more classes is obvious (e.g., `h3` for a 3-class mixture). The appropriate number of latent classes may be determined by comparing AIC for the fitted models¹.

At this time you are advised not to fit more than 2 classes in `secr` because there are technical difficulties with the link function for `pmix`. This defaults to

¹score tests (e.g. McCrea, R. S. and Morgan, B. J. T. (2011) Multistate mark-recapture model selection using score tests *Biometrics* 67, 234–241) are not appropriate because the models are not nested, at least that’s how it seems to me

`mlogit` (after the ‘`mlogit`’ link in MARK), and in fact any attempt to change the link is ignored.

On the bright side, it is unlikely that you will ever have enough data to support more than 2 classes. For the data in the example above, the 2-class and 3-class models have identical log likelihood to 4 decimal places, while the latter requires 2 extra parameters to be estimated (this is to be expected as the data were simulated from a null model with no heterogeneity).

Notes

It’s worth mentioning a perennial issue of interpretation: Do the latent classes have biological reality? The answer is No. Fitting a finite mixture model does not require or imply that there is a matching structure in the population (discrete types of animal). A mixture model is merely a convenient way to capture heterogeneity.

When more than one real parameter is modelled as a mixture, there is an ambiguity: is the population split once into latent classes common to all real parameters, or is the population split separately for each real parameter? The second option would require a distinct level of the mixing parameter for each real parameter. `secr` implements only the ‘common classes’ option, which saves one parameter.

References

- Borchers, D. L. and Efford, M. G. (2008) Spatially explicit maximum likelihood methods for capture–recapture studies. *Biometrics* **64**, 377–385.
- Pledger, S. (2000) Unified maximum likelihood estimates for closed capture–recapture models using mixtures. *Biometrics* **56**, 434–442.