# Package 'disprofas'

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Type Package
Title Non-Parametric Dissolution Profile Analysis

Version 0.1.3

**Description** Similarity of dissolution profiles is assessed using the similarity factor f2 according to the EMA guideline (European Medicines Agency 2010) "On the investigation of bioequivalence". Dissolution profiles are regarded as similar if the f2 value is between 50 and 100. For the applicability of the similarity factor f2, the variability between profiles needs to be within certain limits. Often, this constraint is violated. One possibility in this situation is to resample the measured profiles in order to obtain a bootstrap estimate of f2 (Shah et al. (1998) <doi:10.1023/A:1011976615750>). Other alternatives are the model-independent non-parametric multivariate confidence region (MCR) procedure (Tsong et al. (1996) <doi:10.1177/009286159603000427>) or the T2-test for equivalence procedure (Hoffelder (2016) <a href="https://www.ecv.de/suse\_item.php?suseId=Z|pi|8430">https://www.ecv.de/suse\_item.php?suseId=Z|pi|8430</a>). Functions for estimation of f1, f2, bootstrap f2, MCR / T2-test for equivalence procedure are implemented.

**License** GPL (>= 2)

URL https://github.com/piusdahinden/disprofas

BugReports https://github.com/piusdahinden/disprofas/issues

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bootstrap\_f2

Bootstrap f2

# Description

The function bootstrap\_f2() generates R bootstrap replicates of the similarity factor  $f_2$  based on resampling of complete profiles (nonparametric bootstrap) or on resampling per time point the values between profiles (parametric bootstrap). Estimates of "normal", "basic", "student", "percent" and of "bias-corrected, accelerated" (BCa) percentile intervals are returned.

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### Usage

```
bootstrap_f2(
  data,
  tcol,
  grouping,
  rand_mode = "complete",
  R = 999,
  each = 12,
  new_seed = 100,
  confid = 0.9,
  use_EMA = "no",
  bounds = c(1, 85),
  ...
)
```

### **Arguments**

data A data frame with the dissolution profile data in wide format.

A vector of indices specifying the columns in data that contain the % release

values. The length of tcol must be three or longer.

grouping A character string specifying the column in data that contains the group names

(i.e. a factorial variable, e.g., for the differentiation of batches or formulations

of a drug product).

rand\_mode A character string indicating if complete profiles shall be randomised ("complete",

the default) or individual data points ("individual").

R An integer specifying the number of bootstrap replicates. The default is 999.

each An integer specifying the number of dissolution profiles to be selected per group

per randomisation round. The default is 12.

new\_seed An integer for setting the seed for random number generation. The default is

100.

confid A numeric value between 0 and 1 specifying the confidence limit for the calcu-

lation of the bootstrap confidence intervals. The default is 0.9.

use\_EMA A character string indicating if the similarity factor  $f_2$  should be calculated ac-

cording to the EMA guideline "On the investigation of bioequivalence" ("yes") or not ("no", the default). The default is "no" because the bootstrap  $f_2$  method is one of the possible solutions if the condition concerning the variability between the profiles does not allow the evaluation of  $f_2$  according to the EMA guideline. A third option is "ignore". If use\_EMA is "yes" or "no" the appropriate profile portion is determined on the basis of the values of the parameter bounds. If it is

"ignore", the complete profiles are used as specified by the parameter tcol.

A numeric vector of the form c(lower, upper) specifying the "lower" and "upper" limits, respectively, for the % drug release given that use\_EMA is "no". The

default is c(1, 85). Mean % release values of any of the two groups being compared that are smaller than or equal to the lower bound are ignored and only the first mean % release value that is greater than or equal to the upper bound is

first mean % release value that is greater than or equal to the upper bound is

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included while all the subsequent values are ignored. If use\_EMA is "yes" the bounds are c(1, 85) per definition.

.. Named parameters of the functions stat.fun(), ran.fun() and boot().

#### **Details**

Information on  $f_2$  can be found in at least three FDA guidances and in the guideline of the European Medicines Agency (EMA) "On the investigation of bioequivalence" (EMA 2010). For the assessment of the similarity of dissolution profiles using the similarity factor  $f_2$  according to the EMA guideline the following constraints do apply:

- 1. A minimum of three time points (without zero) are necessary.
- 2. The time points should be the same for the two formulations.
- 3. For every time point and for each formulation at least 12 data points are required.
- 4. A maximum of one mean value per formulation may be > 85% dissolved.
- 5. The coefficient of variation (%CV) should be < 20% for the first time point and < 10% from the second to the last time point for any formulation.

Dissolution profiles are regarded as similar if the  $f_2$  value is between 50 and 100.

One often encountered problem is that the %CV constraint cannot be fulfilled. One possibility in this situation is the use of the bootstrap  $f_2$  method (Shah 1998) by which the distribution of  $f_2$  is simulated to obtain an unbiased estimate of the expected value of  $f_2$  and the variability of the underlying distribution. For the  $f_2$  calculation only those parts of the profiles are taken into account where the means (per formulation) are > d% dissolved (e.g., d=1) and a maximum of one mean value per formulation is > 85% dissolved. In the literature it is suggested to make use of the lower 90% bias corrected and accelerated (BCa) confidence interval (CI) limit to come to a decision in terms of similarity (Stevens (2015)).

#### Value

An object of class 'bootstrap\_f2' is returned, containing the following list elements:

function from the 'boot' package.

Boot	An object of class 'boot' with the corresponding components.
Profile.TP	A named numeric vector of the columns in data specified by tcol and depending on the selection of use_EMA. Given that the column names contain extractable numeric information, e.g., specifying the testing time points of the dissolution profile, it contains the corresponding values. Elements where no numeric information could be extracted are NA.
L	A vector of the Jackknife leave-one-out-values.
CI	An object of class 'bootci' which contains the intervals.
BCa_CI	The lower and upper limits of the BCa interval calculated by the boot.ci()

ShahBCa\_CI The lower and upper limits of the BCa interval calculated according to Shah (Shah 1998).

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#### References

United States Food and Drug Administration (FDA). Guidance for industry: dissolution testing of immediate release solid oral dosage forms. 1997.

https://www.fda.gov/media/70936/download

United States Food and Drug Administration (FDA). Guidance for industry: immediate release solid oral dosage form: scale-up and post-approval changes, chemistry, manufacturing and controls, *in vitro* dissolution testing, and *in vivo* bioequivalence documentation (SUPAC-IR). 1995.

https://www.fda.gov/media/70949/download

European Medicines Agency (EMA), Committee for Medicinal Products for Human Use (CHMP). Guideline on the Investigation of Bioequivalence. 2010; CPMP/EWP/QWP/1401/98 Rev. 1.

https://www.ema.europa.eu/en/documents/scientific-guideline/guideline-investigation-bioequivalence-en.pdf

Stevens, R. E., Gray, V., Dorantes, A., Gold, L., and Pham, L. Scientific and regulatory standards for assessing product performance using the similarity factor,  $f_2$ . AAPS Journal. 2015; **17**(2): 301-306.

doi: 10.1208/s122480159723y

Shah, V. P., Tsong, Y., Sathe, P., and Liu, J. P. *In vitro* dissolution profile comparison - statistics and analysis of the similarity factor,  $f_2$ . *Pharm Res.* 1998; **15**(6): 889-896.

doi: 10.1023/A:1011976615750

# See Also

get\_jackknife\_values, boot, boot.ci, mimcr, mztia.

dip1

Dissolution data of a reference and a test batch

#### **Description**

A data set containing the dissolution data of one reference batch and one test batch of n=6 tablets each, i.e. the dissolution profiles of the % drug release observed within a period of 120 minutes.

# Usage

data(dip1)

#### **Format**

A data frame with 12 observations and 10 variables:

**type** Factor with levels R (Reference) and T (Test)

tablet Factor with levels 1 to 6 representing individual tablets

**t.5** Numeric of the % release at the 5 minutes testing point

t.10 Numeric of the % release at the 10 minutes testing point

**t.15** Numeric of the % release at the 15 minutes testing point

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- **t.20** Numeric of the % release at the 20 minutes testing point
- **t.30** Numeric of the % release at the 30 minutes testing point
- **t.60** Numeric of the % release at the 60 minutes testing point
- t.90 Numeric of the % release at the 90 minutes testing point
- **t.120** Numeric of the % release at the 120 minutes testing point

#### Source

See reference: Example data set shown in Table 1.

#### References

Tsong, Y., Hammerstrom, T., Sathe, P.M., and Shah, V.P. Statistical assessment of mean differences between two dissolution data sets. *Drug Inf J.* 1996; **30**: 1105-1112. doi: 10.1177/009286159603000427

# **Examples**

str(dip1)

dip2

Dissolution data of one reference batch and five test batches

# **Description**

A data set containing the dissolution data of one reference batch and five test batches of n=12 tablets each, i.e. the dissolution profiles of the % drug release observed within a period of 180 minutes.

# Usage

data(dip2)

# **Format**

A data frame with 72 observations and 8 variables:

type Factor with levels Reference and Test

tablet Factor with levels 1 to 12 representing individual tablets

batch Factor with levels b0, b1, b2, b3, b4 and b5

**t.0** Numeric of the % release at the initial testing point

t.30 Numeric of the % release at the 30 minutes testing point

**t.60** Numeric of the % release at the 60 minutes testing point

t.90 Numeric of the % release at the 90 minutes testing point

t.180 Numeric of the % release at the 180 minutes testing point

*dip3* 7

# Source

See reference: Example data set shown in Table 4.

#### References

Shah, V. P., Tsong, Y., Sathe, P., and Liu, J. P. *In vitro* dissolution profile comparison - statistics and analysis of the similarity factor,  $f_2$ . *Pharm Res.* 1998; **15**(6): 889-896. doi: 10.1023/A:1011976615750

# **Examples**

str(dip2)

dip3

Dissolution data of two different capsule formulations

# **Description**

A data set containing the dissolution data of one reference batch and one test batch of n=12 capsules each, i.e. the dissolution profiles of the % drug release observed at 15, 20 and 25 minutes.

# Usage

data(dip3)

#### **Format**

A data frame with 24 observations and 6 variables:

cap Factor with levels 1 to 12 representing individual capsules

**batch** Factor with levels white and blue representing the colours of two different capsule formulations

**type** Factor with levels ref (Reference) and test (Test)

x.15 Numeric of the % release at the 15 minutes testing point

x.20 Numeric of the % release at the 20 minutes testing point

x.25 Numeric of the % release at the 25 minutes testing point

#### **Source**

See reference: Example data set shown in Table 1. Data set 'ex\_data\_JoBS' from package 'T2EQ'.

# References

Hoffelder, T., Goessl, R., and Wellek, S. Multivariate equivalence tests for use in pharmaceutical development. *J Biopharm Stat* (2015) **25**(3): 417-437.

doi: 10.1080/10543406.2014.920344

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# **Examples**

```
str(dip3)
if (requireNamespace("T2EQ")) {
library(T2EQ)

data(ex_data_JoBS, envir = environment())
    str(ex_data_JoBS)
    rm(ex_data_JoBS)
}
```

dip4

Dissolution data of two different formulations

# Description

A data set containing the dissolution data of one reference batch and one test batch of n=12 items each, i.e. the dissolution profiles of the % drug release observed at 10, 20 and 30 minutes.

# Usage

```
data(dip4)
```

# **Format**

A data frame with 24 observations and 2 variables:

type Factor with levels ref (Reference) and test (Test)

x.10 Numeric of the % release at the 10 minutes testing point

**x.20** Numeric of the % release at the 20 minutes testing point

**x.30** Numeric of the % release at the 30 minutes testing point

### **Source**

See reference: Example data set underlying Figure 1. Data set 'ex\_data\_pharmind' from package 'T2EQ'.

# References

Hoffelder, T. Highly variable dissolution profiles. Comparison of  $T^2$ -test for equivalence and  $f_2$  based methods. *Pharm Ind.* 2016; **78**(4): 587-592.

```
https://www.ecv.de/suse_item.php?suseId=Z|pi|8430
```

dip5

# **Examples**

```
str(dip4)

if (requireNamespace("T2EQ")) {
  library(T2EQ)

  data(ex_data_pharmind, envir = environment())
  str(ex_data_pharmind)
  rm(ex_data_pharmind)
}
```

dip5

Fluid weights of drink cans

# Description

The response values of this data set correspond to the values published in the SAS/QC(R) 13.1 (2013) User's Guide, Chapter 5 (The CAPABILITY Procedure). The data set is described on page 199: The fluid weights of 100 drink cans were measured in ounces. The filling process is assumed to be in statistical control.

# Usage

```
data(dip5)
```

# **Format**

A data frame with 100 observations and 3 variables:

```
type Factor with the single level referencebatch Factor with levels b1 to b100weight Weight of drink cans
```

# **Source**

See reference: Chapter 5 (The CAPABILITY Procedure), Cans data set shown on page 199.

# References

```
SAS Institute Inc. 2013. SAS/QC(R) 13.1 User's Guide. Cary, NC: SAS Institute Inc. https://support.sas.com/documentation/cdl/en/qcug/66857/PDF/default/qcug.pdf
```

# **Examples**

```
str(dip5)
```

Dissimilarity factor f1 for dissolution data

# f1

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# **Description**

The function f1() calculates the dissimilarity factor  $f_1$ .

# Usage

```
f1(data, tcol, grouping, use_EMA = "yes", bounds = c(1, 85))
```

# **Arguments**

data A data frame with the dissolution profile data in wide format.

tcol A vector of indices specifying the columns in data that contain the % release

values. The length of tcol must be three or longer.

grouping A character string specifying the column in data that contains the group names

(i.e. a factorial variable, e.g., for the differentiation of batches or formulations

f1

of a drug product).

use\_EMA A character string indicating if the dissimilarity factor  $f_1$  should be calculated

following the EMA guideline "On the investigation of bioequivalence" ("yes", the default) or not ("no"), i.e. the recommendations concerning the similarity factor  $f_2$ . A third option is "ignore". If use\_EMA is "yes" or "no" the appropriate profile portion is determined on the basis of the values of the parameter bounds. If it is "ignore", the complete profiles are used as specified by the

parameter tcol.

bounds A numeric vector of the form c(lower, upper) specifying the "lower" and "up-

per" limits, respectively, for the % drug release given that use\_EMA is "no". The default is c(1, 85). Mean % release values of any of the two groups being compared that are smaller than or equal to the lower bound are ignored and only the first mean % release value that is greater than or equal to the upper bound is included while all the subsequent values are ignored. If use\_EMA is "yes" the

bounds are c(1, 85) per definition.

### **Details**

Similarity of dissolution profiles is often assessed using the similarity factor  $f_2$ , as recommended by the EMA guideline (European Medicines Agency 2010) "On the investigation of bioequivalence". The evaluation of the similarity factor is based on the following constraints:

- 1. A minimum of three time points (zero excluded).
- 2. The time points should be the same for the two formulations.
- 3. Twelve individual values for every time point for each formulation.
- 4. Not more than one mean value of > 85% dissolved for any of the formulations.

f1 11

5. The relative standard deviation or coefficient of variation of any product should be less than 20% for the first time point and less than 10% from the second to the last time point.

The dissimilarity factor, or difference factor,  $f_1$ , is the counterpart of the similarity factor  $f_2$ . The difference factor  $f_1$  is a measure of the relative error between two curves. Current FDA guidelines suggest that two profiles can be considered similar if  $f_1$  is less than  $15 \ (0-15)$  and  $f_2$  is greater than  $50 \ (50-100)$ , which is equivalent to an average difference of 10% at all sampling time points. The dissimilarity factor  $f_1$  is calculated by aid of the equation

$$f_1 = 100 \times \frac{\sum_{t=1}^{n} (|\bar{R}(t) - \bar{T}(t)|)}{\sum_{t=1}^{n} (\bar{R}(t))}.$$

In this equation

 $f_1$  is the dissimilarity factor,

n is the number of time points,

 $\bar{R}(t)$  is the mean percent reference drug dissolved at time t after initiation of the study, and

 $\bar{T}(t)$  is the mean percent test drug dissolved at time t after initiation of the study.

#### Value

A list with the following elements is returned:

f1 A numeric value representing the similarity factor  $f_1$ .

Profile.TP

A named numeric vector of the columns in data specified by tcol and depending on the selection of use\_EMA. Given that the column names contain extractable numeric information, e.g., specifying the testing time points of the dissolution profile, it contains the corresponding values. Elements where no numeric information could be extracted are NA.

#### References

United States Food and Drug Administration (FDA). Guidance for industry: dissolution testing of immediate release solid oral dosage forms. 1997.

https://www.fda.gov/media/70936/download

United States Food and Drug Administration (FDA). Guidance for industry: immediate release solid oral dosage form: scale-up and post-approval changes, chemistry, manufacturing and controls, *in vitro* dissolution testing, and *in vivo* bioequivalence documentation (SUPAC-IR). 1995.

https://www.fda.gov/media/70949/download

European Medicines Agency (EMA), Committee for Medicinal Products for Human Use (CHMP). Guideline on the Investigation of Bioequivalence. 2010; CPMP/EWP/QWP/1401/98 Rev. 1.

https://www.ema.europa.eu/en/documents/scientific-guideline/guideline-investigation-bioequivalence-en.pdf

# See Also

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# **Examples**

```
\# Dissolution data of one reference batch and one test batch of n = 6
# tablets each:
str(dip1)
# 'data.frame': 12 obs. of 10 variables:
# $ type : Factor w/ 2 levels "R", "T": 1 1 1 1 1 1 2 2 2 2 ...
# $ tablet: Factor w/ 6 levels "1","2","3","4",..: 1 2 3 4 5 6 1 2 3 4 ...
# $ t.5 : num 42.1 44.2 45.6 48.5 50.5 ...
# $ t.10 : num 59.9 60.2 55.8 60.4 61.8 ...
# $ t.15 : num 65.6 67.2 65.6 66.5 69.1 ...
# $ t.20 : num 71.8 70.8 70.5 73.1 72.8 ...
# $ t.30 : num 77.8 76.1 76.9 78.5 79 ...
# $ t.60 : num 85.7 83.3 83.9 85 86.9 ...
# $ t.90 : num 93.1 88 86.8 88 89.7 ...
# $ t.120 : num 94.2 89.6 90.1 93.4 90.8 ...
# Use of defaults, i.e. 'use_EMA = "yes"', 'bounds = c(1, 85)'
# Comparison always involves only two groups.
f1(data = dip1, tcol = 3:10, grouping = "type")
# $f1
# [1] 18.19745
# $Profile.TP
# t.5 t.10 t.15 t.20 t.30 t.60 t.90
# 5 10 15 20 30 60 90
# Use of 'use_EMA = "no"', 'bounds = c(5, 80)'
f1(data = dip1, tcol = 3:10, grouping = "type", use_EMA = "no",
  bounds = c(5, 80))
# $f1
# [1] 21.333
# $Profile.TP
# t.5 t.10 t.15 t.20 t.30 t.60
# 5 10 15 20 30 60
# Use of 'use_EMA = "no"', 'bounds = c(1, 95)'
f1(data = dip1, tcol = 3:10, grouping = "type", use_EMA = "no",
  bounds = c(1, 95)
# $f1
# [1] 16.22299
# $Profile.TP
# t.5 t.10 t.15 t.20 t.30 t.60 t.90 t.120
       10 15
                    20
                         30
                               60
                                    90 120
# In this case, the whole profiles are used. The same result is obtained
# when setting 'use_EMA = "ignore" (ignoring values passed to 'bounds').
```

Similarity factor f2 for dissolution data

# Description

f2

The function f2() calculates the similarity factor  $f_2$ .

#### **Usage**

```
f2(data, tcol, grouping, use_EMA = "yes", bounds = c(1, 85))
```

# **Arguments**

data A data frame with the dissolution profile data in wide format.

tcol A vector of indices specifying the columns in data that contain the % release

values. The length of tcol must be three or longer.

grouping A character string specifying the column in data that contains the group names

(i.e. a factorial variable, e.g., for the differentiation of batches or formulations

of a drug product).

use\_EMA A character string indicating if the similarity factor  $f_2$  should be calculated fol-

lowing the EMA guideline "On the investigation of bioequivalence" ("yes", the default) or not ("no"). A third option is "ignore". If use\_EMA is "yes" or "no" the appropriate profile portion is determined on the basis of the values of the parameter bounds. If it is "ignore", the complete profiles are used as specified

by the parameter tcol.

bounds A numeric vector of the form c(lower, upper) specifying the "lower" and "up-

per" limits, respectively, for the % drug release given that use\_EMA is "no". The default is c(1, 85). Mean % release values of any of the two groups being compared that are smaller than or equal to the lower bound are ignored and only the first mean % release value that is greater than or equal to the upper bound is included while all the subsequent values are ignored. If use\_EMA is "yes" the

bounds are c(1, 85) per definition.

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# **Details**

Similarity of dissolution profiles is assessed using the similarity factor  $f_2$  according to the EMA guideline (European Medicines Agency 2010) "On the investigation of bioequivalence". The evaluation of the similarity factor is based on the following constraints:

- 1. A minimum of three time points (zero excluded).
- 2. The time points should be the same for the two formulations.
- 3. Twelve individual values for every time point for each formulation.
- 4. Not more than one mean value of > 85% dissolved for any of the formulations.
- 5. The relative standard deviation or coefficient of variation of any product should be less than 20% for the first time point and less than 10% from the second to the last time point.

The similarity factor  $f_2$  is calculated by aid of the equation

$$f_2 = 50 \times \log \left( \frac{100}{\sqrt{1 + \frac{\sum_{t=1}^{n} (\bar{R}(t) - \bar{T}(t))^2}{n}}} \right).$$

In this equation

 $f_2$  is the similarity factor,

n is the number of time points,

 $\bar{R}(t)$  is the mean percent reference drug dissolved at time t after initiation of the study, and

T(t) is the mean percent test drug dissolved at time t after initiation of the study.

Dissolution profiles are regarded as similar if the  $f_2$  value is between 50 and 100.

### Value

A list with the following elements is returned:

f2 A numeric value representing the similarity factor  $f_2$ .

Profile.TP A named numeric vector of the columns in data specified by tcol and depending on the selection of use\_EMA. Given that the column names contain extractable numeric information, e.g., specifying the testing time points of the dissolution profile, it contains the corresponding values. Elements where no numeric information could be extracted are NA.

#### References

United States Food and Drug Administration (FDA). Guidance for industry: dissolution testing of immediate release solid oral dosage forms. 1997.

https://www.fda.gov/media/70936/download

United States Food and Drug Administration (FDA). Guidance for industry: immediate release solid oral dosage form: scale-up and post-approval changes, chemistry, manufacturing and controls, *in* 

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*vitro* dissolution testing, and *in vivo* bioequivalence documentation (SUPAC-IR). 1995. https://www.fda.gov/media/70949/download

European Medicines Agency (EMA), Committee for Medicinal Products for Human Use (CHMP). Guideline on the Investigation of Bioequivalence. 2010; CPMP/EWP/QWP/1401/98 Rev. 1.

https://www.ema.europa.eu/en/documents/scientific-guideline/guideline-investigation-bioequivalence-en.pdf

#### See Also

f1.

# **Examples**

```
\# Dissolution data of one reference batch and one test batch of n = 6
# tablets each:
str(dip1)
# 'data.frame': 12 obs. of 10 variables:
# $ type : Factor w/ 2 levels "R", "T": 1 1 1 1 1 1 2 2 2 2 ...
# $ tablet: Factor w/ 6 levels "1", "2", "3", "4", ...: 1 2 3 4 5 6 1 2 3 4 ...
        : num 42.1 44.2 45.6 48.5 50.5 ...
# $ t.10 : num 59.9 60.2 55.8 60.4 61.8 ...
# $ t.15 : num 65.6 67.2 65.6 66.5 69.1 ...
# $ t.20 : num 71.8 70.8 70.5 73.1 72.8 ...
# $ t.30 : num 77.8 76.1 76.9 78.5 79 ...
# $ t.60 : num 85.7 83.3 83.9 85 86.9 ...
# $ t.90 : num 93.1 88 86.8 88 89.7 ...
# $ t.120 : num 94.2 89.6 90.1 93.4 90.8 ...
# Use of defaults, i.e. 'use_EMA = "yes"', 'bounds = c(1, 85)'
# Comparison always involves only two groups.
f2(data = dip1, tcol = 3:10, grouping = "type")
# $f2
# [1] 40.83405
# $Profile.TP
# t.5 t.10 t.15 t.20 t.30 t.60 t.90
# 5 10 15 20 30 60 90
# Use of 'use_EMA = "no"', 'bounds = c(5, 80)'
f2(data = dip1, tcol = 3:10, grouping = "type", use_EMA = "no",
  bounds = c(5, 80)
# $f2
# [1] 39.24385
# $Profile.TP
# t.5 t.10 t.15 t.20 t.30 t.60
# 5 10 15 20 30 60
# Use of 'use_EMA = "no"', 'bounds = c(1, 95)'
```

```
f2(data = dip1, tcol = 3:10, grouping = "type", use_EMA = "no",
  bounds = c(1, 95)
# $f2
# [1] 42.11197
# $Profile.TP
# t.5 t.10 t.15 t.20 t.30 t.60 t.90 t.120
        10
              15
                     20
                           30
                                       90
                                           120
                                 60
# In this case, the whole profiles are used. The same result is obtained
# when setting 'use_EMA = "ignore"' (ignoring values passed to 'bounds').
f2(data = dip1, tcol = 3:10, grouping = "type", use_EMA = "ignore")
# Passing in a data frame with a grouping variable with a number of levels that
# differs from two produces an error.
tmp <- rbind(dip1,</pre>
             data.frame(type = "T2",
                        tablet = as.factor(1:6),
                        dip1[7:12, 3:10]))
tryCatch(
 f2(data = tmp, tcol = 3:10, grouping = "type"),
 error = function(e) message(e),
 finally = message("\nMaybe you want to remove unesed levels in data."))
# Error in f1(data = tmp, tcol = 3:10, grouping = "type") :
   The number of levels in column type differs from 2.
```

gep\_by\_nera

Points on confidence region (found by Newton-Raphson search)

#### **Description**

The function  $gep\_by\_nera()$  is a function for finding points on confidence region (CR) bounds by aid of the "Method of Lagrange Multipliers" (MLM) and by "Newton-Raphson" (nera) optimisation. The multivariate confidence interval for profiles with four time points, e.g., is an "ellipse" in four dimensions.

#### **Usage**

```
gep_by_nera(n_p, K, mean_diff, S_pool, F_crit, y, max_trial, tol)
```

# **Arguments**

Κ

n\_p A positive integer specifying the number of (time) points  $n_p$ .

A non-negative numeric value specifying the scaling factor K for the calculation of the Hotelling's  $T^2$  statistic.

mean_diff	A vector of the mean differences between the dissolution profiles of the reference and the test batch. It must have the length specified by the parameter $n_p$ .
S_pool	The pooled variance-covariance matrix of the dissolution profiles of the reference and the test batch. It must have the dimension $n_p \times n_p$ .
F_crit	The critical $F$ value (i.e. a non-negative numeric).
У	A numeric vector of $y$ values that serve as starting points for the Newton-Raphson search, i.e. values supposed to lie on or close to the confidence interval bounds. It must have a length of $n_p+1$ .
max_trial	A positive integer specifying the maximum number of Newton-Raphson search rounds to be performed.
tol	A non-negative numeric specifying the accepted minimal difference between two consecutive search rounds.

#### **Details**

The function gep\_by\_nera() determines the points on the CR bounds for each of the  $n_p$  time points. It does so by aid of the "Method of Lagrange Multipliers" (MLM) and by "Newton-Raphson" (nera) optimisation, as proposed by Margaret Connolly (Connolly 2000).

For more information, see the sections "Comparison of highly variable dissolution profiles" and "Similarity limits in terms of MSD" below.

#### Value

A list with the following elements is returned:

points	A matrix with one column and $n_p+1$ rows is returned, where rows 1 to $n_p$ represent, for each time point, the points on the $CR$ bounds. For symmetry reasons, the points on the opposite side are obtained by addition/subtraction. The last row in the matrix, with index $n_p+1$ , represents the $\lambda$ parameter of the MLM, also known as $lambda\ multiplier\ method$ , that is used to optimise under constraint(s). The variable $\lambda$ is thus called the $Lagrange\ multiplier$ .
converged	A logical stating if the NR algorithm converged or not.
n.trial	Number of trials until convergence.
max.trial	Maximal number of trials.

# Comparison of highly variable dissolution profiles

When comparing the dissolution data of a post-approval change product and a reference approval product, the goal is to assess the similarity between the mean dissolution values at the observed sample time points. A widely used method is the  $f_2$  method that was introduced by Moore & Flanner (1996). Similarity testing criteria based on  $f_2$  can be found in several FDA guidelines and in the guideline of the European Medicines Agency (EMA) "On the investigation of bioequivalence" (EMA 2010).

In situations where within-batch variation is greater than 15%, FDA guidelines recommend use of a multivariate confidence interval as an alternative to the  $f_2$  method. This can be done using the following stepwise procedure:

1. Establish a similarity limit in terms of "Multivariate Statistical Distance" (MSD) based on inter-batch differences in % drug release from reference (standard approved) formulations, i.e. the so- called "Equivalence Margin" (EM).

- 2. Calculate the MSD between test and reference mean dissolutions.
- 3. Estimate the 90% confidence interval (CI) of the true MSD as determined in step 2.
- 4. Compare the upper limit of the 90% CI with the similarity limit determined in step 1. The test formulation is declared to be similar to the reference formulation if the upper limit of the 90% CI is less than or equal to the similarity limit.

# Similarity limits in terms of MSD

For the calculation of the "Multivariate Statistical Distance" (MSD), the procedure proposed by Tsong et al. (1996) can be considered as well-accepted method that is actually recommended by the FDA. According to this method, a multivariate statistical distance, called Mahalanobis distance, is used to measure the difference between two multivariate means. This distance measure is calculated as

$$D_M = \sqrt{\left(oldsymbol{x_T} - oldsymbol{x_R}
ight)^{ op} oldsymbol{S}_{pooled}^{-1}\left(oldsymbol{x_T} - oldsymbol{x_R}
ight)},$$

where  $S_{pooled} = \frac{(S_T + S_R)}{2}$  is the sample variance-covariance matrix pooled across the batches,  $x_T$  and  $x_R$  are the vectors of the sample means for the test (T) and reference (R) profiles, and  $S_T$  and  $S_R$  are the variance-covariance matrices of the test and reference profiles.

In order to determine the similarity limits in terms of the MSD, i.e. the Mahalanobis distance between the two multivariate means of the dissolution profiles of the formulations to be compared, Tsong et al. (1996) proposed using the equation

$$D_M^{max} = \sqrt{oldsymbol{d}_g^{ op} oldsymbol{S}_{pooled}^{-1} oldsymbol{d}_g},$$

where  $d_g$  is a  $1 \times p$  vector with all p elements equal to an empirically defined limit  $d_g$ , e.g., 15%, for the maximum tolerable difference at all time points, and p is the number of sampling points. By assuming that the data follow a multivariate normal distribution, the 90% confidence region (CR) bounds for the true difference between the mean vectors,  $\mu_T - \mu_R$ , can be computed for the resultant vector  $\mu$  to satisfy the following condition:

$$CR = K \left( \boldsymbol{\mu} - (\boldsymbol{x_T} - \boldsymbol{x_R}) \right)^{\top} S_{pooled}^{-1} \left( \boldsymbol{\mu} - (\boldsymbol{x_T} - \boldsymbol{x_R}) \right) \le F_{p,n_T + n_R - p - 1,0.9},$$

where K is the scaling factor that is calculated as

$$K = \frac{n_T n_R}{n_T + n_R} \cdot \frac{n_T + n_R - p - 1}{(n_T + n_R - 2) \cdot p},$$

and  $F_{p,n_T+n_R-p-1,0.9}$  is the  $90^{th}$  percentile of the F distribution with degrees of freedom p and  $n_T+n_R-p-1$ . It is obvious that  $(n_T+n_R)$  must be greater than (p+1). The formula for CR gives a p-variate 90% confidence region for the possible true differences.

#### References

United States Food and Drug Administration (FDA). Guidance for industry: dissolution testing of immediate release solid oral dosage forms. 1997.

```
https://www.fda.gov/media/70936/download
```

United States Food and Drug Administration (FDA). Guidance for industry: immediate release solid oral dosage form: scale-up and post-approval changes, chemistry, manufacturing and controls, *in vitro* dissolution testing, and *in vivo* bioequivalence documentation (SUPAC-IR). 1995.

```
https://www.fda.gov/media/70949/download
```

European Medicines Agency (EMA), Committee for Medicinal Products for Human Use (CHMP). Guideline on the Investigation of Bioequivalence. 2010; CPMP/EWP/QWP/1401/98 Rev. 1.

https://www.ema.europa.eu/en/documents/scientific-guideline/guideline-investigation-bioequivalence-en.pdf

Moore, J.W., and Flanner, H.H. Mathematical comparison of curves with an emphasis on *in-vitro* dissolution profiles. *Pharm Tech.* 1996; **20**(6): 64-74.

Tsong, Y., Hammerstrom, T., Sathe, P.M., and Shah, V.P. Statistical assessment of mean differences between two dissolution data sets. *Drug Inf J.* 1996; **30**: 1105-1112.

```
doi: 10.1177/009286159603000427
```

Connolly, M. SAS(R) IML Code to calculate an upper confidence limit for multivariate statistical distance; 2000; Wyeth Lederle Vaccines, Pearl River, NY.

```
https://analytics.ncsu.edu/sesug/2000/p-902.pdf
```

#### See Also

```
mimcr, bootstrap_f2.
```

# **Examples**

```
# Collecting the required information
time_points <- suppressWarnings(as.numeric(gsub("([^0-9])", "",</pre>
                                                       colnames(dip1))))
tico <- which(!is.na(time_points))</pre>
b1 <- dip1$type == "R"
n_tp <- length(tico)</pre>
n_b1 <- length(dip1[b1, "type"])</pre>
n_b2 <- length(dip1[!b1, "type"])</pre>
df_b1 \leftarrow n_tp
df_b2 < - n_b1 + n_b2 - n_tp - 1
K_{limit} \leftarrow (n_b2 * n_b1) / (n_b2 + n_b1)
K \leftarrow K_{limit} * df_b2 / (df_b1 * (n_b2 + n_b1 - 2))
mean_b1 <- apply(X = dip1[b1, tico], MARGIN = 2, FUN = mean)</pre>
mean_b2 \leftarrow apply(X = dip1[!b1, tico], MARGIN = 2, FUN = mean)
mean_diff <- mean_b2 - mean_b1</pre>
S_b1 \leftarrow cov(dip1[b1, tico])
S_b2 <- cov(dip1[!b1, tico])</pre>
S \leftarrow ((n_b1 - 1) * S_b1 + (n_b2 - 1) * S_b2) / (n_b1 + n_b2 - 2)
```

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```
F_{crit} \leftarrow qf(p = (1 - 0.05), df1 = df_b1, df2 = df_b2)
y_b1 < -rep(1, times = (n_tp + 1))
# Calling gep_by_nera()
res <- gep_by_nera(n_p = n_{tp}, K = K, mean_diff = mean_diff, S_{pool} = S,
                   F_{crit} = F_{crit}, y = y_b1, max_{trial} = 100, tol = 1e-9)
# Expected result in res[["points"]]
               [,1]
# t.5 -15.7600077
# t.10 -13.6501734
# t.15 -11.6689469
# t.20
         -9.8429369
# t.30
         -6.6632182
# t.60
         -0.4634318
# t.90
          2.2528551
# t.120 3.3249569
        -17.6619995
# Rows t.5 to t.120 represent the points on the CR bounds.The unnamed last row
# represents the Lagrange multiplier lambda.
# If 'max_trial' is too small, the Newton-Raphson search may not converge.
tryCatch(
  gep_by_nera(n_p = n_tp, K = K, mean_diff = mean_diff, S_pool = S,
              F_{crit} = F_{crit}, y = y_b1, max_{trial} = 5, tol = 1e-9),
  warning = function(w) message(w),
  finally = message("\nMaybe increasing the number of max_trial could help."))
```

get\_hotellings

Hotelling's statistics (for two independent (small) samples)

# **Description**

The function  $get\_hotellings()$  estimates the parameters for Hotelling's two-sample  $T^2$  statistic for small samples.

### Usage

```
get_hotellings(m1, m2, signif)
```

# **Arguments**

m1 A matrix with the data of the reference group.

m2 A matrix with the same dimensions as matrix m1, with the data of the test group.

A positive numeric value between 0 and 1 specifying the significance level. The default value is 0.05.

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# **Details**

The two-sample Hotelling's  $T^2$  test statistic is given by

$$T^2 = \left(ar{oldsymbol{x}}_1 - ar{oldsymbol{x}}_2
ight)^ op \left(oldsymbol{S}_p\left(rac{1}{n_1} + rac{1}{n_2}
ight)
ight)^{-1} \left(ar{oldsymbol{x}}_1 - ar{oldsymbol{x}}_2
ight).$$

For large samples, this test statistic will be approximately chi-square distributed with p degrees of freedom. However, this approximation does not take into account the variation due to the variance-covariance matrix estimation. Therefore, Hotelling's  $T^2$  statistic is transformed into an F-statistic using the expression

$$F = \frac{n_1 + n_2 - p - 1}{(n_1 + n_2 - 2)p}T^2,$$

where  $n_1$  and  $n_2$  are the sample sizes of the two samples being compared and p is the number of variables.

Under the null hypothesis,  $H_0: \mu_1 = \mu_2$ , this F-statistic will be F-distributed with p and  $n_1 + n_2 - p$  degrees of freedom.  $H_0$  is rejected at significance level  $\alpha$  if the F-value exceeds the critical value from the F-table evaluated at  $\alpha$ , i.e.  $F > F_{p,n_1+n_2-p-1,\alpha}$ . The null hypothesis is satisfied if, and only if, the population means are identical for all variables. The alternative is that at least one pair of these means is different.

The following assumptions concerning the data are made:

- The data from population i is a sample from a population with mean vector  $\mu_i$ . In other words, it is assumed that there are no sub-populations.
- The data from both populations have common variance-covariance matrix  $\Sigma$ .
- The subjects from both populations are independently sampled.
- Both populations are normally distributed.

#### Value

A list with the following elements is returned:

Parameters	Parameters determined for the estimation of Hotelling's $\mathbb{T}^2$ .
S.pool	Pooled variance-covariance matrix.
covs	A list with the elements $S.b1$ and $S.b2$ , i.e. the variance-covariance matrices of the reference and the test group, respectively.
means	A list with the elements mean.b1, mean.b2 and mean.diff, i.e. the average profile values (for each time point) of the reference and the test group and the corresponding differences of the averages, respectively.

The Parameters element contains the following information:

DM	Mahalanobis distance of the samples.
df1	Degrees of freedom (number of variables or time points).
df2	Degrees of freedom (number of rows - number of variables - 1)

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alpha	Provided significance level.
K	Scaling factor for $F$ to account for the distribution of the ${\cal T}^2$ statistic.
k	Scaling factor for the squared Mahalanobis distance to obtain the ${\cal T}^2$ statistic.
T2	Hotelling's $T^2$ statistic ( $F$ -distributed).
F	Observed $F$ value.
F.crit	Critical $F$ value.
p.F	$p$ value for Hotelling's $T^2$ test statistic.

#### References

Hotelling, H. The generalisation of Student's ratio. Ann Math Stat. 1931; 2(3): 360-378.

Hotelling, H. (1947) *Multivariate quality control illustrated by air testing of sample bombsights*. In: Eisenhart, C., Hastay, M.W., and Wallis, W.A., Eds., Techniques of Statistical Analysis, McGraw Hill, New York, 111-184.

#### See Also

```
mimcr, get_sim_lim.
```

# **Examples**

```
\# Dissolution data of one reference batch and one test batch of n = 6
# tablets each:
str(dip1)
# 'data.frame': 12 obs. of 10 variables:
# $ type : Factor w/ 2 levels "R", "T": 1 1 1 1 1 1 2 2 2 2 ...
# $ tablet: Factor w/ 6 levels "1", "2", "3", "4", ...: 1 2 3 4 5 6 1 2 3 4 ...
# $ t.5 : num 42.1 44.2 45.6 48.5 50.5 ...
# $ t.10 : num 59.9 60.2 55.8 60.4 61.8 ...
# $ t.15 : num 65.6 67.2 65.6 66.5 69.1 ...
# $ t.20 : num 71.8 70.8 70.5 73.1 72.8 ...
# $ t.30 : num 77.8 76.1 76.9 78.5 79 ...
# $ t.60 : num 85.7 83.3 83.9 85 86.9 ...
# $ t.90 : num 93.1 88 86.8 88 89.7 ...
# $ t.120 : num 94.2 89.6 90.1 93.4 90.8 ...
# Estimation of the parameters for Hotelling's two-sample T2 statistic
# (for small samples)
res <-
  get_hotellings(m1 = as.matrix(dip1[dip1$type == "R", c("t.15", "t.90")]),
                 m2 = as.matrix(dip1[dip1$type == "T", c("t.15", "t.90")]),
                 signif = 0.1)
res$S.pool
res$Parameters
# Expected results in res$S.pool
                 t.90
          t.15
# t.15 3.395808 1.029870
```

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```
# t.90 1.029870 4.434833

# Expected results in res$Parameters
# DM df1 df2 signif K
# 1.044045e+01 2.000000e+00 9.000000e+00 1.000000e-01 1.350000e+00
# k T2 F F.crit p.F
# 3.000000e+00 3.270089e+02 1.471540e+02 3.006452e+00 1.335407e-07
```

get\_sim\_lim Similarity limit

# **Description**

The function get\_sim\_lim() estimates a similarity limit in terms of the "Multivariate Statistical Distance" (MSD).

# Usage

```
get_sim_lim(mtad, lhs)
```

# Arguments

mtad	A numeric value specifying the "maximum tolerable average difference" (MTAD) of the profiles of two formulations at all time points (in %). The default value is 10. It determines the size of the similarity limit $d_g$ .
lhs	A list of the estimates of Hotelling's two-sample $T^2$ statistic for small samples as returned by the function get_hotellings().

# **Details**

Details about the estimation of similarity limits in terms of the "Multivariate Statistical Distance" (MSD) are explained in the corresponding section below.

# Value

A vector containing the following information is returned:

DM	The Mahalanobis distance of the samples.
df1	Degrees of freedom (number of variables or time points).
df2	Degrees of freedom (number of rows - number of variables - 1).
alpha	The provided significance level.
K	Scaling factor for $F$ to account for the distribution of the $\mathbb{T}^2$ statistic.
k	Scaling factor for the squared Mahalanobis distance to obtain the $\mathcal{T}^2$ statistic.
T2	Hotelling's $T^2$ statistic ( $F$ -distributed).
F	Observed $F$ value.
ncp.Hoffelder	Non-centrality parameter for calculation of the $F$ statistic ( $T^2$ test procedure).

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F. crit Critical F value (Tsong's procedure).

F.crit.Hoffelder

Critical F value ( $T^2$  test procedure).

p. F The p value for the Hotelling's  $T^2$  test statistic.

p.F.Hoffelder The p value for the Hotelling's  $T^2$  statistic based on the non-central F distribu-

tion.

MTAD Specified "maximum tolerable average difference" (MTAD) of the profiles of

two formulations at each individual time point (in %).

Sim.Limit Critical Mahalanobis distance or similarity limit (Tsong's procedure).

# Similarity limits in terms of MSD

For the calculation of the "Multivariate Statistical Distance" (MSD), the procedure proposed by Tsong et al. (1996) can be considered as well-accepted method that is actually recommended by the FDA. According to this method, a multivariate statistical distance, called Mahalanobis distance, is used to measure the difference between two multivariate means. This distance measure is calculated as

$$D_{M} = \sqrt{\left(\boldsymbol{x_{T}} - \boldsymbol{x_{R}}\right)^{\top} \boldsymbol{S}_{pooled}^{-1} \left(\boldsymbol{x_{T}} - \boldsymbol{x_{R}}\right)},$$

where  $S_{pooled} = \frac{(S_T + S_R)}{2}$  is the sample variance-covariance matrix pooled across the batches,  $x_T$  and  $x_R$  are the vectors of the sample means for the test (T) and reference (R) profiles, and  $S_T$  and  $S_R$  are the variance-covariance matrices of the test and reference profiles.

In order to determine the similarity limits in terms of the MSD, i.e. the Mahalanobis distance between the two multivariate means of the dissolution profiles of the formulations to be compared, Tsong et al. (1996) proposed using the equation

$$D_M^{max} = \sqrt{oldsymbol{d}_g^{ op} oldsymbol{S}_{pooled}^{-1} oldsymbol{d}_g},$$

where  $d_g$  is a  $1 \times p$  vector with all p elements equal to an empirically defined limit  $d_g$ , e.g., 15%, for the maximum tolerable difference at all time points, and p is the number of sampling points. By assuming that the data follow a multivariate normal distribution, the 90% confidence region (CR) bounds for the true difference between the mean vectors,  $\mu_T - \mu_R$ , can be computed for the resultant vector  $\mu$  to satisfy the following condition:

$$CR = K (\mu - (x_T - x_R))^{\top} S_{pooled}^{-1} (\mu - (x_T - x_R)) \le F_{p,n_T + n_R - p - 1,0.9},$$

where K is the scaling factor that is calculated as

$$K = \frac{n_T n_R}{n_T + n_R} \cdot \frac{n_T + n_R - p - 1}{(n_T + n_R - 2) \cdot p},$$

and  $F_{p,n_T+n_R-p-1,0.9}$  is the  $90^{th}$  percentile of the F distribution with degrees of freedom p and  $n_T+n_R-p-1$ . It is obvious that  $(n_T+n_R)$  must be greater than (p+1). The formula for CR gives a p-variate 90% confidence region for the possible true differences.

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# T2 test for equivalence

Based on the distance measure for profile comparison that was suggested by Tsong et al. (1996), i.e. the Mahalanobis distance, Hoffelder (2016) proposed a statistical equivalence procedure for that distance, the so-called  $T^2$  test for equivalence (T2EQ). It is used to demonstrate that the Mahalanobis distance between reference and test group dissolution profiles is smaller than the "Equivalence Margin" (EM). Decision in favour of equivalence is taken if the p value of this test statistic is smaller than the pre-specified significance level  $\alpha$ , i.e. if  $p < \alpha$ . The p value is calculated by aid of the formula

$$p = F_{p,n_T + n_R - p - 1, ncp, \alpha} \frac{n_T + n_R - p - 1}{(n_T + n_R - 2)p} T^2,$$

where  $\alpha$  is the significance level and ncp is the so-called "non-centrality parameter" that is calculated by

$$\frac{n_T n_R}{n_T + n_R} \left( D_M^{max} \right)^2.$$

The test statistic being used is Hotelling's  $T^2$  that is given as

$$T^2 = \frac{n_T n_R}{n_T + n_R} \left( \boldsymbol{x_T} - \boldsymbol{x_R} \right)^{\top} \boldsymbol{S}_{pooled}^{-1} \left( \boldsymbol{x_T} - \boldsymbol{x_R} \right).$$

As mentioned elsewhere,  $d_g$  is a  $1 \times p$  vector with all p elements equal to an empirically defined limit  $d_g$ . Thus, the components of the vector  $d_g$  can be interpreted as upper bound for a kind of "average" allowed difference between test and reference profiles, the "global similarity limit". Since the EMA requires that "similarity acceptance limits should be pre-defined and justified and not be greater than a 10% difference", it is recommended to use 10%, not 15% as proposed by Tsong et al. (1996), for the maximum tolerable difference at all time points.

#### References

Tsong, Y., Hammerstrom, T., Sathe, P.M., and Shah, V.P. Statistical assessment of mean differences between two dissolution data sets. *Drug Inf J.* 1996; **30**: 1105-1112.

doi: 10.1177/009286159603000427

Wellek S. (2010) *Testing statistical hypotheses of equivalence and noninferiority* (2nd ed.). Chapman & Hall/CRC, Boca Raton.

doi: 10.1201/EBK1439808184

Hoffelder, T. Highly variable dissolution profiles. Comparison of  $T^2$ -test for equivalence and  $f_2$  based methods. *Pharm Ind.* 2016; **78**(4): 587-592.

https://www.ecv.de/suse\_item.php?suseId=Z|pi|8430

#### See Also

mimcr, get\_hotellings.

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# **Examples**

```
\# Dissolution data of one reference batch and one test batch of n = 6
# tablets each:
str(dip1)
# 'data.frame': 12 obs. of 10 variables:
# $ type : Factor w/ 2 levels "R", "T": 1 1 1 1 1 1 2 2 2 2 ...
# $ tablet: Factor w/ 6 levels "1","2","3","4",...: 1 2 3 4 5 6 1 2 3 4 ...
# $ t.5 : num 42.1 44.2 45.6 48.5 50.5 ...
# $ t.10 : num 59.9 60.2 55.8 60.4 61.8 ...
# $ t.15 : num 65.6 67.2 65.6 66.5 69.1 ...
# $ t.20 : num 71.8 70.8 70.5 73.1 72.8 ...
# $ t.30 : num 77.8 76.1 76.9 78.5 79 ...
# $ t.60 : num 85.7 83.3 83.9 85 86.9 ...
# $ t.90 : num 93.1 88 86.8 88 89.7 ...
# $ t.120 : num 94.2 89.6 90.1 93.4 90.8 ...
# Estimation of the parameters for Hotelling's two-sample T2 statistic
# (for small samples)
hs \leftarrow get_hotellings(m1 = as.matrix(dip1[dip1$type == "R", c("t.15", "t.90")]),
                   m2 = as.matrix(dip1[dip1$type == "T", c("t.15", "t.90")]),
                   signif = 0.1)
# Estimation of the similarity limit in terms of the "Multivariate Statistical
# Distance" (MSD)for a "maximum tolerable average difference" (mtad) of 10
res <- get_sim_lim(mtad = 15, hs)</pre>
# Expected results in res
                            df1
                                             df2
            DM
                                                           alpha
  1.044045e+01
                   2.000000e+00
                                    9.000000e+00
                                                    1.000000e-01
#
#
           K
                            k
                                             T2
# 1.350000e+00
                3.000000e+00
                                    3.270089e+02
                                                    1.471540e+02
# ncp.Hoffelder
                    F.crit F.crit.Hoffelder
                                                             p.F
                3.006452e+00
# 2.782556e+02
                                 8.357064e+01
                                                    1.335407e-07
# p.F.Hoffelder
                   MTAD
                                     Sim.Limit
# 4.822832e-01 1.500000e+01
                                    9.630777e+00
```

mimcr

Model-independent multivariate confidence region (MIMCR) procedure

# Description

The function mimcr() assesses the equivalence of highly variable dissolution profiles. It does so by applying different methods proposed in the literature, implementing the non-parametric "Model-Independent Multivariate Confidence Region" (MIMCR) procedure and the " $T^2$  test for equivalence" of dissolution data as proposed by Hoffelder (2016).

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# Usage

```
mimcr(
  data,
  tcol,
  grouping,
  fit_n_obs = FALSE,
 mtad = 10,
  signif = 0.05,
 max_trial = 50,
 bounds = c(1, 85),
  tol = 1e-09
)
```

# **Arguments**

data A data frame with the dissolution profile data in wide format.

tcol A vector of indices specifying the columns in data that contain the % release

values. The length of tcol must be three or longer.

grouping A character string specifying the column in data that contains the group names (i.e. a factorial variable, e.g., for the differentiation of batches or formulations

of a drug product).

fit\_n\_obs A logical value specifying if the number of rows per level in the column specified

by the grouping parameter should be adjusted to be equal given that they are not equal. The default is FALSE because each group should have the same number of observations. If fit\_n\_obs is TRUE, redundant observations from the level with more observations are dropped, i.e. only the observations 1 to the number of observations of the level with the fewer observations will be used for the

comparison of the two groups.

mtad A numeric value specifying the "maximum tolerable average difference" (MTAD)

of the profiles of two formulations at all time points (in %). The default value is

10. It determines the size of the similarity limit  $d_q$ .

signif A positive numeric value between 0 and 1 specifying the significance level for

the calculation of the "Confidence Region" (CR). The coverage of CR is (1 -

signif)100%. The default value is 0.05.

A positive integer specifying the maximum number of Newton-Raphson search max\_trial

rounds to be performed.

bounds A numeric vector of the form c(lower, upper) specifying the "lower" and "up-

> per" limits, respectively, for the % drug release. The default is c(1, 85). Mean % release values of any of the two groups being compared that are smaller than or equal to the lower bound are ignored and only the first mean % release value that is greater than or equal to the upper bound is included while all the subse-

quent values are ignored.

A non-negative numeric specifying the accepted minimal difference between

two consecutive search rounds.

tol

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#### **Details**

The function mimcr() assesses the equivalence of highly variable dissolution profiles by aid of a "Model-Independent Multivariate Confidence Region" (MIMCR) procedure as proposed by Tsong et al. (1996) and by aid of a "T2 test for equivalence" as proposed by Hoffelder (2016).

For details see the sections "Comparison of highly variable dissolution profiles", "Similarity limits in terms of MSD" and "T2 test for equivalence" below.

#### Value

An object of class 'mimcr' is returned, containing the following list elements:

Similarity Conclusion concerning similarity.

Parameters Parameters calculated during the assessment.

NR.CI List with results from the Newton-Raphson (NR) search.

Profile.TP A named numeric vector of the columns in data specified by tcol. Given that

the column names contain extractable numeric information, e.g., specifying the testing time points of the dissolution profile, it contains the corresponding val-

ues. Elements where no numeric information could be extracted are NA.

The Parameters element contains the following information:

DM The Mahalanobis distance of the samples.

df1 Degrees of freedom (number of variables or time points).

df2 Degrees of freedom (number of rows - number of variables - 1).

alpha The provided significance level.

K Scaling factor for F to account for the distribution of the  $T^2$  statistic.

k Scaling factor for the squared Mahalanobis distance to obtain the  $T^2$  statistic.

T2 Hotelling's  $T^2$  statistic (F-distributed).

F Observed F value.

ncp. Hoffelder Non-centrality parameter for calculation of the F statistic ( $T^2$  test procedure).

F. crit Critical F value (Tsong's procedure).

F.crit.Hoffelder

Critical F value ( $T^2$  test procedure).

p.F The p value for the Hotelling's  $T^2$  test statistic.

p.F.Hoffelder The p value for the Hotelling's  $T^2$  statistic based on the non-central F distribu-

tion.

MTAD Specified "maximum tolerable average difference" (MTAD) of the profiles of

two formulations at each individual time point (in %).

Sim. Limit Critical Mahalanobis distance or similarity limit (Tsong's procedure).

Obs. U Observed lower limit (Tsong's procedure).

Obs. U Observed upper limit (Tsong's procedure).

The NR.CI element contains the following information:

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CI A matrix of the points on the CR bounds for each time point. converged A logical specifying if the NR algorithm converged or not. n.trial Number of trials until convergence.

Maximal number of trials.

Warning A warning message, if applicable, or otherwise NULL.

Error An error message, if applicable, or otherwise NULL.

# Comparison of highly variable dissolution profiles

When comparing the dissolution data of a post-approval change product and a reference approval product, the goal is to assess the similarity between the mean dissolution values at the observed sample time points. A widely used method is the  $f_2$  method that was introduced by Moore & Flanner (1996). Similarity testing criteria based on  $f_2$  can be found in several FDA guidelines and in the guideline of the European Medicines Agency (EMA) "On the investigation of bioequivalence" (EMA 2010).

In situations where within-batch variation is greater than 15%, FDA guidelines recommend use of a multivariate confidence interval as an alternative to the  $f_2$  method. This can be done using the following stepwise procedure:

- 1. Establish a similarity limit in terms of "Multivariate Statistical Distance" (MSD) based on inter-batch differences in % drug release from reference (standard approved) formulations, i.e. the so- called "Equivalence Margin" (EM).
- 2. Calculate the MSD between test and reference mean dissolutions.
- 3. Estimate the 90% confidence interval (CI) of the true MSD as determined in step 2.
- 4. Compare the upper limit of the 90% CI with the similarity limit determined in step 1. The test formulation is declared to be similar to the reference formulation if the upper limit of the 90% CI is less than or equal to the similarity limit.

#### Similarity limits in terms of MSD

For the calculation of the "Multivariate Statistical Distance" (MSD), the procedure proposed by Tsong et al. (1996) can be considered as well-accepted method that is actually recommended by the FDA. According to this method, a multivariate statistical distance, called Mahalanobis distance, is used to measure the difference between two multivariate means. This distance measure is calculated as

$$D_{M} = \sqrt{\left(\boldsymbol{x_{T}} - \boldsymbol{x_{R}}\right)^{\top} \boldsymbol{S}_{pooled}^{-1} \left(\boldsymbol{x_{T}} - \boldsymbol{x_{R}}\right)},$$

where  $S_{pooled} = \frac{(S_T + S_R)}{2}$  is the sample variance-covariance matrix pooled across the batches,  $x_T$  and  $x_R$  are the vectors of the sample means for the test (T) and reference (R) profiles, and  $S_T$  and  $S_R$  are the variance-covariance matrices of the test and reference profiles.

In order to determine the similarity limits in terms of the MSD, i.e. the Mahalanobis distance between the two multivariate means of the dissolution profiles of the formulations to be compared, Tsong et al. (1996) proposed using the equation

$$D_M^{max} = \sqrt{\boldsymbol{d}_g^{ op} \boldsymbol{S}_{pooled}^{-1} \boldsymbol{d}_g},$$

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where  $d_g$  is a  $1 \times p$  vector with all p elements equal to an empirically defined limit  $d_g$ , e.g., 15%, for the maximum tolerable difference at all time points, and p is the number of sampling points. By assuming that the data follow a multivariate normal distribution, the 90% confidence region (CR) bounds for the true difference between the mean vectors,  $\mu_T - \mu_R$ , can be computed for the resultant vector  $\mu$  to satisfy the following condition:

$$CR = K (\mu - (x_T - x_R))^{\top} S_{pooled}^{-1} (\mu - (x_T - x_R)) \le F_{p,n_T + n_R - p - 1,0.9},$$

where K is the scaling factor that is calculated as

$$K = \frac{n_T n_R}{n_T + n_R} \cdot \frac{n_T + n_R - p - 1}{(n_T + n_R - 2) \cdot p},$$

and  $F_{p,n_T+n_R-p-1,0.9}$  is the  $90^{th}$  percentile of the F distribution with degrees of freedom p and  $n_T+n_R-p-1$ . It is obvious that  $(n_T+n_R)$  must be greater than (p+1). The formula for CR gives a p-variate 90% confidence region for the possible true differences.

# T2 test for equivalence

Based on the distance measure for profile comparison that was suggested by Tsong et al. (1996), i.e. the Mahalanobis distance, Hoffelder (2016) proposed a statistical equivalence procedure for that distance, the so-called  $T^2$  test for equivalence (T2EQ). It is used to demonstrate that the Mahalanobis distance between reference and test group dissolution profiles is smaller than the "Equivalence Margin" (EM). Decision in favour of equivalence is taken if the p value of this test statistic is smaller than the pre-specified significance level  $\alpha$ , i.e. if  $p < \alpha$ . The p value is calculated by aid of the formula

$$p = F_{p,n_T + n_R - p - 1, ncp, \alpha} \frac{n_T + n_R - p - 1}{(n_T + n_R - 2)p} T^2,$$

where  $\alpha$  is the significance level and ncp is the so-called "non-centrality parameter" that is calculated by

$$\frac{n_T n_R}{n_T + n_R} \left( D_M^{max} \right)^2.$$

The test statistic being used is Hotelling's  $T^2$  that is given as

$$T^{2} = \frac{n_{T} n_{R}}{n_{T} + n_{R}} \left( \boldsymbol{x_{T}} - \boldsymbol{x_{R}} \right)^{\top} \boldsymbol{S}_{pooled}^{-1} \left( \boldsymbol{x_{T}} - \boldsymbol{x_{R}} \right).$$

As mentioned elsewhere,  $d_g$  is a  $1 \times p$  vector with all p elements equal to an empirically defined limit  $d_g$ . Thus, the components of the vector  $d_g$  can be interpreted as upper bound for a kind of "average" allowed difference between test and reference profiles, the "global similarity limit". Since the EMA requires that "similarity acceptance limits should be pre-defined and justified and not be greater than a 10% difference", it is recommended to use 10%, not 15% as proposed by Tsong et al. (1996), for the maximum tolerable difference at all time points.

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#### References

United States Food and Drug Administration (FDA). Guidance for industry: dissolution testing of immediate release solid oral dosage forms. 1997.

```
https://www.fda.gov/media/70936/download
```

United States Food and Drug Administration (FDA). Guidance for industry: immediate release solid oral dosage form: scale-up and post-approval changes, chemistry, manufacturing and controls, *in vitro* dissolution testing, and *in vivo* bioequivalence documentation (SUPAC-IR). 1995.

```
https://www.fda.gov/media/70949/download
```

European Medicines Agency (EMA), Committee for Medicinal Products for Human Use (CHMP). Guideline on the Investigation of Bioequivalence. 2010; CPMP/EWP/QWP/1401/98 Rev. 1.

https://www.ema.europa.eu/en/documents/scientific-guideline/guideline-investigation-bioequivalence-en.pdf

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```
doi: 10.1177/009286159603000427
```

Tsong, Y., Hammerstrom, T., and Chen, J.J. Multipoint dissolution specification and acceptance sampling rule based on profile modeling and principal component analysis. *J Biopharm Stat.* 1997; **7**(3): 423-439.

```
doi: 10.1080/10543409708835198
```

Wellek S. (2010) Testing statistical hypotheses of equivalence and noninferiority (2nd ed.). Chapman & Hall/CRC, Boca Raton.

```
doi: 10.1201/EBK1439808184
```

Hoffelder, T. Highly variable dissolution profiles. Comparison of  $T^2$ -test for equivalence and  $f_2$  based methods. *Pharm Ind.* 2016; **78**(4): 587-592.

```
https://www.ecv.de/suse_item.php?suseId=Z|pi|8430
```

#### See Also

```
gep_by_nera, bootstrap_f2, mztia.
```

# **Examples**

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```
res1$Similarity
res1$Parameters
# Expected results in res1$Similarity
     Tsong Hoffelder
# "Similar" "Similar"
# Expected results in res1$Parameters
            DM
                            df1
                                              df2
                                                             alpha
#
                    3.000000e+00
                                     2.000000e+01
                                                      5.000000e-02
#
  2.384023e-01
#
             Κ
                              k
                                               T2
                                                                 F
# 1.818182e+00
                    6.000000e+00
                                     3.410141e-01
                                                      1.033376e-01
# ncp.Hoffelder
                          F.crit F.crit.Hoffelder
                                                               p.F
# 3.032296e+01
                    3.098391e+00
                                     4.899274e+00
                                                      9.571526e-01
# p.F.Hoffelder
                            MTAD
                                        Sim.Limit
                                                             0bs.L
  2.890827e-08
                   1.000000e+01
                                     2.248072e+00
                                                      1.067015e+00
#
         Obs.U
 1.543820e+00
# Comparison with T2-test for equivalence for dissolution data from the 'T2EQ'
# package:
if (requireNamespace("T2EQ")) {
 library(T2EQ)
 data(ex_data_JoBS)
 {\tt T2EQ.dissolution.profiles.hoffelder(}\\
   X = as.matrix(dip3[dip3$type == "ref", c("x.15", "x.20", "x.25")]),
   Y = as.matrix(dip3[dip3$type == "test", c("x.15", "x.20", "x.25")]))
}
# Excerpt of output:
# Hotelling's T2:
                                        0.3410141
# Noncentrality parameter:
                                             30.32296
# Significance level:
                                           0.05
                                          0.1033376
# Teststatistic:
# Quantile of noncent. F-distribution:
                                              4.899274
# p-value of the T2-test for equivalence: p = 2.890827e-08
\# Dissolution data of one reference batch and one test batch of n = 6
# tablets each:
str(dip1)
# 'data.frame': 12 obs. of 10 variables:
# $ type : Factor w/ 2 levels "R","T": 1 1 1 1 1 1 2 2 2 2 ...
# $ tablet: Factor w/ 6 levels "1","2","3","4",...: 1 2 3 4 5 6 1 2 3 4 ...
# $ t.5
        : num 42.1 44.2 45.6 48.5 50.5 ...
# $ t.10 : num 59.9 60.2 55.8 60.4 61.8 ...
# $ t.15 : num 65.6 67.2 65.6 66.5 69.1 ...
# $ t.20 : num 71.8 70.8 70.5 73.1 72.8 ...
         : num 77.8 76.1 76.9 78.5 79 ...
# $ t.30
# $ t.60 : num 85.7 83.3 83.9 85 86.9 ...
# $ t.90 : num 93.1 88 86.8 88 89.7 ...
# $ t.120 : num 94.2 89.6 90.1 93.4 90.8 ...
```

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```
# Use of 'bounds = c(1, 85)'
res2 <- mimcr(data = dip1, tcol = 3:10, grouping = "type", bounds = c(1, 85))
res2$Similarity
res2$Profile.TP
res2[["Parameters"]][c("p.F.Hoffelder", "Sim.Limit", "Obs.U")]
# Expected results in res2$Similarity
       Tsong Hoffelder
# "Dissimilar" "Dissimilar"
# Expected results in res2$Profile.TP
# t.5 t.10 t.15 t.20 t.30 t.60 t.90
# 5 10 15 20 30 60 90
# Expected results in
# res2[["Parameters"]][c("p.F.Hoffelder", "Sim.Limit", "Obs.U")]
# p.F.Hoffelder
                Sim.Limit
                                   Obs. U
      0.740219
                11.328041
                                31.679020
# Allow for a larger maximum tolerable average difference (MTAD), e.g., 15.
res3 <- mimcr(data = dip1, tcol = 3:10, grouping = "type", mtad = 15,
            bounds = c(1, 85))
res3$Similarity
res3[["Parameters"]][c("p.F.Hoffelder", "Sim.Limit", "Obs.U")]
# Expected results in res3$Similarity
# Tsong Hoffelder
# "Dissimilar" "Dissimilar"
# Expected results in
# res3[["Parameters"]][c("p.F.Hoffelder", "Sim.Limit", "Obs.U")]
# p.F.Hoffelder Sim.Limit Obs.U
     0.3559019 16.9920622 31.6790198
# Use default 'mtad' but set 'signif = 0.1' and use 'bounds = c(1, 95)' so that
# the complete profiles are taken into account.
res4 <- mimcr(data = dip1, tcol = 3:10, grouping = "type", mtad = 10,
             signif = 0.1, bounds = c(1, 95))
res4$Similarity
res4$Profile.TP
res4[["Parameters"]][c("p.F.Hoffelder", "Sim.Limit", "Obs.U")]
# Expected results in res3$Similarity
       Tsong Hoffelder
# "Dissimilar" "Dissimilar"
# Expected results in res2$Profile.TP
# t.5 t.10 t.15 t.20 t.30 t.60 t.90 t.120
# 5 10 15 20
                       30
                             60 90 120
# Expected results in
# res2[["Parameters"]][c("p.F.Hoffelder", "Sim.Limit", "Obs.U")]
```

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```
# p.F.Hoffelder
                   Sim.Limit
                                      Obs.U
     0.1449045
                   19.4271898
                                 33.3180044
# If 'max_trial' is too small, the Newton-Raphson search may not converge.
tryCatch(
 mimcr(data = dip1, tcol = 3:10, grouping = "type", max_trial = 5),
 warning = function(w) message(w),
 finally = message("\nMaybe increasing the number of max_trial could help."))
# If 'tol' is too big, the points found by the Newton-Raphson search may not
# be located on the confidence region boundary.
tryCatch(
 mimcr(data = dip3, tcol = 4:6, grouping = "batch", tol = 1),
 warning = function(w) message(w),
 finally = message("\nMaybe making tol smaller could help."))
# Passing in a data frame with a grouping variable with a number of levels that
# differs from two produces an error.
tmp <- rbind(dip1,
            data.frame(type = "T2",
                        tablet = as.factor(1:6),
                        dip1[7:12, 3:10]))
tryCatch(
 mimcr(data = tmp, tcol = 3:10, grouping = "type", bounds = c(1, 85)),
 error = function(e) message(e),
 finally = message("\nMaybe you want to remove unesed levels in data."))
# Error in mimcr(data = tmp, tcol = 3:10, grouping = "type", bounds = , :
  The number of levels in column type differs from 2.
```

mztia

Martinez & Zhao Tolerance Interval Approach

# **Description**

The Martinez & Zhao Tolerance Interval Approach (mztia) is a simple approach for the comparison of dissolution profiles. The mztia() function calculates tolerance intervals (TI) at each time point of the dissolution profiles of a set of reference batches. By aid of a graphical display the test batches are checked to lie within the TI boundaries or within certain limits exceeding the TI boundaries by a specified percentage.

# Usage

```
mztia(
  data,
  shape,
  tcol,
  grouping,
```

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```
reference,
response = NULL,
alpha = 0.05,
P = 0.99,
cap = TRUE,
bounds = c(0, 100),
QS = c(5, 15),
...
```

# **Arguments**

data

A data frame with the dissolution profile data in wide or in long format (see parameter shape). If the data frame is in wide format, it is tried to extract the information on the time points of dissolution testing from the column names of the columns specified by the tcol parameter. Thus, they must contain extractable numeric information, e.g., (t\_0, t\_5, t\_10). If the data frame is in long format, it must have a column of time points (column specified via the tcol parameter).

shape

A character string specifying if the data frame is in long or in wide format.

tcol

If shape is "wide" an integer vector of indices, if shape is "long" an integer, specifying the column(s) containing the profile time points. If the data frame is in wide format it is reshaped using the function reshape() from the 'stats' package.

grouping

A character string specifying the column in data that contains the group names (i.e. a factorial variable, e.g., for the differentiation of batches or formulations of a drug product).

reference

A character string specifying the name of the reference group from the grouping variable.

response

A character string that is expected if data is provided in long format in order to specify the column with the % drug release values. The default is NULL.

alpha

A numeric value between 0 and 1 specifying the probability level. The default is 0.05.

Р

A numeric value between 0 and 1 specifying the proportion of the population being enclosed by the tolerance interval boundaries. The default is 0.99.

сар

A logical variable specifying if the calculated tolerance limits should be limited (i.e. *capped*). The default is TRUE.

bounds

A numeric vector of the form c(lower, upper) specifying the "lower" and "upper" limits, respectively, for the % drug release at which the calculated tolerance interval limits should be capped (see parameter cap. This parameter is only relevant if cap = TRUE. The default is c(0, 100).

QS

A numeric vector of the form c(QS1, QS2) that specifies the allowable deviations from the specifications in percent according to the S1 and S2 acceptance criteria of USP chapter <711> on dissolution. The default is c(5, 15).

. . .

Further arguments passed on to the reshape() from the 'stats' package.

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#### **Details**

The tolerance interval approach proposed by Martinez & Zhao (2018) is a simple approach for the comparison of dissolution profiles. The authors propose to calculate for each time point of a set of reference dissolution profiles a tolerance interval (TI), i.e. intervals containing P% of the population of potential values for reference product at a probability level of alpha/2 per tail (i.e., (1-alpha)100% confidence). Based on these TIs the dissolution profiles of the test batch(es) is (are) compared, i.e. the corresponding data points should lie within the TIs. The TIs are calculated as

$$Y_{utl,ltl} = \bar{Y} \pm k \times s$$

where  $\bar{Y}$  is the average, s is the sample standard deviation, and the factor k is calculated according to Hahn (Hahn & Meeker (1991)), as proposed in Martinez & Zhao (2018).

Since the goal of the comparison is not to confirm simply "statistical sameness" but "product comparability", Martinez & Zhao propose allowing acceptable deviations by utilizing the concepts described by the United States Pharmacopoeia (USP), chapter <711> on dissolution, defining allowable deviations from a set of product specifications (Q). The TIs serve as the target value Q at each sampling time. The allowable deviations about Q are defined by the S1 and S2 acceptance criteria of USP chapter <711> on dissolution:

- 1. The S1 level boundary is defined by  $Q\pm 5\%$  at each time point. For every 12 profiles tested, only one profile is allowed to exceed the S1 bounds.
- 2. The S2 level boundary is defined by  $Q\pm15\%$  at each time point. No observation from any of the test dissolution profiles is allowed to exceed the S2 bounds.

In situations where the reference formulation itself has more than one of twelve observations (profiles) exceeding S1 at one or more time points, additional runs of the reference product must be performed. It is deemed appropriate to use the same values of S1 and S2 across all time points because the high variability associated with the early sampling times is already factored into the TIs.

TI calculation according to Hahn is proposed because it appeared to be more numerically stable and gave more consistent TIs than the TI calculation method proposed by Howe (Howe 1969) when samples were very variable. The reason might be due to the less stringent requirements imposed by Hahn's method with respect to the normality of the data.

### Value

An object of class 'mztia' is returned, containing the following elements:

Variables A list of the variables and the corresponding values.

Limits A data frame of the limits calculated for each time point.

Data A data frame consisting of the provided data, complemented by the calculated

tolerance interval results.

Profile.TP If shape is "wide" a named numeric vector of the columns in data specified

by tcol. Given that the column names contain extractable numeric information, e.g., specifying the testing time points of the dissolution profile, it contains the corresponding values. Elements where no numeric information could be extracted are NA. If shape is "long" it is a numeric value, specifying the column

containing the % release values.

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#### References

Martinez, M.N., and Zhao, X. A simple approach for comparing the *in vitro* dissolution profiles of highly variable drug products: a proposal. *AAPS Journal*. (2018); **20**: 78.

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Howe, W.G. Two-sided tolerance limits for normal populations - some improvements. *J Am Stat Assoc.* (1969); **64**: 610-620.

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Hahn, G.J., and Meeker, W. Q. Statistical intervals: A guide for practitioners. (1991); John Wiley & Sons, New York. Hahn's method is also described in: SAS/QC 13.1: User's Guide. Chapter 5, sub-chapter "Details: INTERVALS Statement", pp 421-424. SAS Institute Inc. 2013. Cary, NC. https://support.sas.com/documentation/cdl/en/qcug/66857/PDF/default/qcug.pdf

U.S. Pharmacopoeia. 2016 U.S. Pharmacopoeia-National Formulary (USP 39 NF 34). Volume 1. Rockville, Md: United States Pharmacopeial Convention, Inc; 2015. <711> Dissolution.

#### See Also

bootstrap\_f2, mimcr.

```
# Fluid weights of 100 drink cans were measured in ounces:
str(dip5)
# 'data.frame': 100 obs. of 3 variables:
# $ type : Factor w/ 1 level "reference": 1 1 1 1 1 1 1 1 1 ...
# $ batch : Factor w/ 100 levels "b1","b10","b100",...: 1 13 24 35 46 57 68 ...
# $ weight: num 12.1 12 12 12 12 ...
# Calculation of tolerance intervals
m_alpha_P \leftarrow matrix(c(rep(c(0.01, 0.05, 0.1), each = 3),
                      1 - rep(c(0.1, 0.05, 0.01), times = 3)),
                    ncol = 2, byrow = FALSE)
11 <-
  apply(m_alpha_P, MARGIN = 1, FUN = function(x)
   mztia(data = dip5, shape = "long", tcol = 1, grouping = "type",
          reference = "reference", response = "weight", alpha = x[1],
          P = x[2], cap = FALSE)[["Data"]][102, "weight"])
ul <-
 apply(m_alpha_P, MARGIN = 1, FUN = function(x)
   mztia(data = dip5, shape = "long", tcol = 1, grouping = "type",
          reference = "reference", response = "weight", alpha = x[1],
          P = x[2], cap = FALSE)[["Data"]][103, "weight"])
# Expected results in ll and ul
rbind(ll, ul)
         [,1]
                 [,2]
                         [,3]
                                   [,4]
                                            [,5]
                                                     [,6]
                                                               [,7]
                                                                        Γ.87
# 11 11.91648 11.8987 11.86395 11.92132 11.90446 11.87152 11.92373 11.90734
# ul 12.10212 12.1199 12.15465 12.09728 12.11414 12.14708 12.09487 12.11126
        [,9]
```

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```
# 11 11.8753
# ul 12.1433
# Use a data frame in wide format
\# Dissolution data of one reference batch and one test batch of n = 6
# tablets each:
str(dip1)
# 'data.frame': 12 obs. of 10 variables:
# $ type : Factor w/ 2 levels "R", "T": 1 1 1 1 1 1 2 2 2 2 ...
# $ tablet: Factor w/ 6 levels "1","2","3","4",..: 1 2 3 4 5 6 1 2 3 4 ...
# $ t.5 : num 42.1 44.2 45.6 48.5 50.5 ...
# $ t.10 : num 59.9 60.2 55.8 60.4 61.8 ...
# $ t.15 : num 65.6 67.2 65.6 66.5 69.1 ...
# $ t.20 : num 71.8 70.8 70.5 73.1 72.8 ...
# $ t.30 : num 77.8 76.1 76.9 78.5 79 ...
# $ t.60 : num 85.7 83.3 83.9 85 86.9 ...
# $ t.90 : num 93.1 88 86.8 88 89.7 ...
# $ t.120 : num 94.2 89.6 90.1 93.4 90.8 ...
# Using the defaults; Limits are capped to the range specified by 'bounds'
res1 <- mztia(data = dip1, shape = "wide", tcol = 3:10, grouping = "type",
             reference = "R")
res1$Limits
# Expected results in res1$Limits
                  LTL
                          UTL S1.LTL
                                            S1.UTL S2.LTL
         Mean
      5 46.77167 27.22641 66.31693 22.22641 71.31693 12.22641 81.31693
     10 60.13333 46.15483 74.11184 41.15483 79.11184 31.15483 89.11184
# 3
    15 67.27500 56.90417 77.64583 51.90417 82.64583 41.90417 92.64583
# 4 20 71.98667 65.44354 78.52979 60.44354 83.52979 50.44354 93.52979
# 6 60 84.81667 77.20275 92.43058 72.20275 97.43058 62.20275 107.43058
     90 89.09333 76.24588 100.00000 71.24588 105.00000 61.24588 115.00000
# 8 120 91.43833 80.29321 100.00000 75.29321 105.00000 65.29321 115.00000
# Without capping of limits to 105%
res2 <- mztia(data = dip1, shape = "wide", tcol = 3:10, grouping = "type",
             reference = "R", cap = FALSE)
res2$Limits
# Expected results in res1$Limits
# Time
         Mean
                 LTL
                          UTL S1.LTL
                                            S1.UTL S2.LTL
                                                              S2.UTL
# 1
      5 46.77167 27.22641 66.31693 22.22641 71.31693 12.22641 81.31693
# 2
     10 60.13333 46.15483 74.11184 41.15483 79.11184 31.15483 89.11184
# 3
     15 67.27500 56.90417 77.64583 51.90417 82.64583 41.90417 92.64583
     20 71.98667 65.44354 78.52979 60.44354 83.52979 50.44354 93.52979
     30 78.07000 69.54259 86.59741 64.54259 91.59741 54.54259 101.59741
     60 84.81667 77.20275 92.43058 72.20275 97.43058 62.20275 107.43058
     90 89.09333 76.24588 101.94079 71.24588 106.94079 61.24588 116.94079
# 8 120 91.43833 80.29321 102.58346 75.29321 107.58346 65.29321 117.58346
# Tolerance intervals are calculated exclusively for the level of the
```

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plot.bootstrap\_f2

Plot of the bootstrap f2 simulation

# **Description**

This is a method for the function plot() for objects of class 'bootstrap\_f2'.

## Usage

```
## S3 method for class 'bootstrap_f2'
plot(x, ...)
```

## **Arguments**

x An object of class 'bootstrap\_f2' returned by the bootstrap\_f2() function.

Further arguments passed to or from other methods or arguments that can be passed down to the plot.boot() function.

#### **Details**

The element Boot of the 'bootstrap\_f2' object that is returned by the function bootstrap\_f2() is an object of class 'boot', generated by the function boot() from the 'boot' package. Thus, the corresponding plot method is used. Arguments to the plot.boot() function can be passed via the ... parameter. In addition to making the plot the function prints the result of Shah's lower 90% BCa confidence interval to the console.

## Value

The 'bootstrap\_f2' object passed to the x parameter is returned invisibly.

## See Also

```
bootstrap_f2, boot, plot.boot, methods.
```

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## **Examples**

```
\# Dissolution data of one reference batch and five test batches of n = 12
# tablets each:
str(dip2)
# 'data.frame': 72 obs. of 8 variables:
# $ type : Factor w/ 2 levels "Reference", "Test": 1 1 1 1 1 1 1 1 1 1 ...
# $ tablet: Factor w/ 12 levels "1", "2", "3", "4", ...: 1 2 3 4 5 6 7 8 9 10 ...
# $ batch : Factor w/ 6 levels "b0", "b1", "b2", ...: 1 1 1 1 1 1 1 1 1 1 ...
         : int 0000000000...
# $ t.30 : num 36.1 33 35.7 32.1 36.1 34.1 32.4 39.6 34.5 38 ...
# $ t.60 : num 58.6 59.5 62.3 62.3 53.6 63.2 61.3 61.8 58 59.2 ...
# $ t.90 : num 80 80.8 83 81.3 72.6 83 80 80.4 76.9 79.3 ...
# $ t.180 : num 93.3 95.7 97.1 92.8 88.8 97.4 96.8 98.6 93.3 94 ...
# Bootstrap assessment of data (two groups) by aid of bootstrap_f2() function
# by using 'rand_mode = "complete"' (the default, randomisation of complete
# profiles)
bs1 <- bootstrap_f2(data = dip2[dip2$batch %in% c("b0", "b4"), ],
                    tcol = 5:8, grouping = "batch", rand_mode = "complete",
                    R = 200, new\_seed = 421, use\_EMA = "no")
pbs1 <- plot(bs1)</pre>
# The plot() function returns the 'plot_mztia' object invisibly.
class(bs1)
class(pbs1)
# Use of 'rand_mode = "individual"' (randomisation per time point)
bs2 <- bootstrap_f2(data = dip2[dip2$batch %in% c("b0", "b4"), ],
                    tcol = 5:8, grouping = "batch", rand_mode = "individual",
                    R = 200, new\_seed = 421, use\_EMA = "no")
plot(bs2)
```

plot.plot\_mztia

Plot of the mztia simulation

## Description

This is a method for the function plot() for objects of class 'plot\_mztia'.

## **Usage**

```
## S3 method for class 'plot_mztia'
plot(x, ...)
```

## **Arguments**

x An object of class 'plot\_mztia' returned by the plot\_mztia() function.

Further arguments passed to or from other methods or arguments that can be passed down to the plot.boot() function.

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#### **Details**

The element Graph of the 'plot\_mztia' object that is returned by the function plot\_mztia() is an object of class 'ggplot', generated by the function ggplot() from the 'ggplot2' package. Thus, the corresponding plot method is used for plotting. Arguments to the ggplot() function can be passed via the ... parameter.

#### Value

The 'plot\_mztia' object passed to the x parameter is returned invisibly.

#### See Also

```
mztia, plot_mztia, ggplot(), methods.
```

```
\# Dissolution data of one reference batch and one test batch of n = 6
# tablets each:
str(dip1)
# 'data.frame': 12 obs. of 10 variables:
# $ type : Factor w/ 2 levels "R", "T": 1 1 1 1 1 1 2 2 2 2 ...
# $ tablet: Factor w/ 6 levels "1","2","3","4",...: 1 2 3 4 5 6 1 2 3 4 ...
# $ t.5 : num 42.1 44.2 45.6 48.5 50.5 ...
# $ t.10 : num 59.9 60.2 55.8 60.4 61.8 ...
# $ t.15 : num 65.6 67.2 65.6 66.5 69.1 ...
# $ t.20 : num 71.8 70.8 70.5 73.1 72.8 ...
# $ t.30 : num 77.8 76.1 76.9 78.5 79 ...
# $ t.60 : num 85.7 83.3 83.9 85 86.9 ...
# $ t.90 : num 93.1 88 86.8 88 89.7 ...
# $ t.120 : num 94.2 89.6 90.1 93.4 90.8 ...
# Assessment of data by aid of the mztia() function
res1 <- mztia(data = dip1, shape = "wide", tcol = 3:10, grouping = "type",
             reference = "R", cap = FALSE)
# The 'mztia' object can be passed on to the plot_mztia() function. This
# function does not produce any output but returns a 'plot_mztia' object.
gg1 <- plot_mztia(res1)</pre>
gg2 <- plot(gg1)
# The plot() function returns the 'plot_mztia' object invisibly.
class(gg1)
class(gg2)
```

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plot\_mztia

Graphical representation of the of MZTIA estimation

## **Description**

The function plot\_mztia() makes a graphical representation of the estimates done by the mztia() function.

## Usage

```
plot_mztia(x, ...)
```

#### **Arguments**

- x An object of class 'mztia' returned by the mztia() function.
- ... Additional parameters that can be passed on to the ggplot() function.

#### **Details**

A graphical representation of the information in the Data element of the object that is returned by mztia() function is made by aid of the ggplot() function from the 'ggplot2' package and added as new list element to the mztia object. Ideally, the data frame provided to the mztia() function allows drawing a time course of the % drug release values. If a single time point is available, the tolerance intervals of the groups specified by the grouping parameter (e.g., for the differentiation of batches or formulations of a drug product) are displayed.

# Value

An object of class 'plot\_mztia' is returned invisibly, consisting of the elements of the 'mztia' object and an additional element named Graph. The element Graph is a 'ggplot' object returned by calling the ggplot() function.

#### See Also

```
mztia, ggplot.
```

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```
# $ t.20 : num 71.8 70.8 70.5 73.1 72.8 ...
# $ t.30 : num 77.8 76.1 76.9 78.5 79 ...
# $ t.60 : num 85.7 83.3 83.9 85 86.9 ...
# $ t.90 : num 93.1 88 86.8 88 89.7 ...
# $ t.120 : num 94.2 89.6 90.1 93.4 90.8 ...
# Analyse the data by aid of the mztia() function.
res1 <- mztia(data = dip1, shape = "wide", tcol = 3:10, grouping = "type",
              reference = "R", cap = FALSE)
# The 'mztia' object can be passed on to the plot_mztia() function. This
# function does not produce any output. It returns a 'plot_mztia' object that
# is essentially an 'mztia' object augmented by a 'ggplot' object.
gg1 <- plot_mztia(res1)</pre>
gg1
# Since the element gg1$Graph is a 'ggplot' object it can be used for further
# manipulation by aid of 'ggplot2' functions.
if (requireNamespace("ggplot2")) {
  library(ggplot2)
  gg1$Graph + labs(title = "Dissolution Data Assessment",
                   x = "Time [min]", y = "Drug Release [%]")
}
# Use a data frame in long format.
# Fluid weights of 100 drink cans were measured in ounces:
str(dip5)
# 'data.frame': 100 obs. of 3 variables:
# $ type : Factor w/ 1 level "reference": 1 1 1 1 1 1 1 1 1 1 ...
# $ batch : Factor w/ 100 levels "b1","b10","b100",...: 1 13 24 35 46 57 68 ...
# $ weight: num 12.1 12 12 12 12 ...
res2 <- mztia(data = dip5, shape = "long", tcol = 3, grouping = "type",
             reference = "reference", response = "weight", cap = FALSE,
             QS = c(5, 15) / 100)
gg2 <- plot_mztia(res2)</pre>
gg2
if (requireNamespace("ggplot2")) {
  library(ggplot2)
  gg2$Graph + labs(title = "Tolerance Intervals",
                   x = NULL, y = "Weight [ounces]")
}
```

44 print.bootstrap\_f2

#### **Description**

This is a method for the function print() for objects of class 'bootstrap\_f2'.

## Usage

```
## S3 method for class 'bootstrap_f2'
print(x, ...)
```

## Arguments

- x An object of class 'bootstrap\_f2' returned by the bootstrap\_f2() function.
- Further arguments passed to or from other methods or arguments that can be passed down to the print.boot() and print.bootci() functions.

#### **Details**

The elements Boot and CI of the 'bootstrap\_f2' object that is returned by the function bootstrap\_f2() are objects of type 'boot' and 'bootci', respectively, generated by the functions boot() and boot.ci(), respectively, from the 'boot' package. Thus, the corresponding print methods are used. Arguments to the print.boot() and print.bootci() functions can be passed via the ... parameter.

#### Value

The 'bootstrap\_f2' object passed to the x parameter is returned invisibly.

#### See Also

```
bootstrap_f2, boot, boot.ci, print.boot, print.bootci, methods.
```

```
# Dissolution data of one reference batch and five test batches of n = 12
# tablets each:
str(dip2)
# 'data.frame': 72 obs. of 8 variables:
# $ type : Factor w/ 2 levels "Reference", "Test": 1 1 1 1 1 1 1 1 1 1 ...
# $ tablet: Factor w/ 12 levels "1","2","3","4",..: 1 2 3 4 5 6 7 8 9 10 ...
# $ batch : Factor w/ 6 levels "b0", "b1", "b2", ...: 1 1 1 1 1 1 1 1 1 1 ...
        : int 00000000000...
# $ t.0
# $ t.30 : num 36.1 33 35.7 32.1 36.1 34.1 32.4 39.6 34.5 38 ...
# $ t.60 : num 58.6 59.5 62.3 62.3 53.6 63.2 61.3 61.8 58 59.2 ...
# $ t.90 : num 80 80.8 83 81.3 72.6 83 80 80.4 76.9 79.3 ...
# $ t.180 : num 93.3 95.7 97.1 92.8 88.8 97.4 96.8 98.6 93.3 94 ...
# Bootstrap assessment of data (two groups) by aid of bootstrap_f2() function
# by using 'rand_mode = "complete"' (the default, randomisation of complete
# profiles)
bs1 <- bootstrap_f2(data = dip2[dip2$batch %in% c("b0", "b4"), ],
```

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```
tcol = 5:8, grouping = "batch", rand_mode = "complete",
                   R = 200, new\_seed = 421, use\_EMA = "no")
# Print of a summary of the assessment
print(bs1)
# STRATIFIED BOOTSTRAP
#
# Call:
   boot(data = data, statistic = get_f2, R = R, strata = data[, grouping],
        grouping = grouping, tcol = tcol[ok])
#
# Bootstrap Statistics :
# original
                bias std. error
# t1* 50.07187 -0.02553234 0.9488015
#
# BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
# Based on 200 bootstrap replicates
# CALL :
   boot.ci(boot.out = t_boot, conf = confid, type = "all", L = jack$loo.values)
#
# Intervals :
# Level
              Normal
                                  Basic
# 90% (48.54, 51.66) (48.46, 51.71)
# Level
           Percentile
                                 BCa
# 90% (48.43, 51.68) (48.69, 51.99)
# Calculations and Intervals on Original Scale
# Some BCa intervals may be unstable
# Shah's lower 90% BCa confidence interval:
# 48.64613
# Use of 'rand_mode = "individual"' (randomisation per time point)
bs2 <- bootstrap_f2(data = dip2[dip2$batch %in% c("b0", "b4"), ],
                    tcol = 5:8, grouping = "batch", rand_mode = "individual",
                   R = 200, new\_seed = 421, use\_EMA = "no")
# Print of a summary of the assessment
print(bs2)
# PARAMETRIC BOOTSTRAP
#
# Call:
   boot(data = data, statistic = get_f2, R = R, sim = "parametric",
        ran.gen = rand_indiv_points, mle = mle, grouping = grouping,
         tcol = tcol[ok], ins = seq_along(b1))
#
```

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```
#
# Bootstrap Statistics :
   original bias std. error
# t1* 50.07187 -0.1215656 0.9535517
# BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
# Based on 200 bootstrap replicates
# CALL :
   boot.ci(boot.out = t_boot, conf = confid, type = "all", L = jack$loo.values)
#
# Intervals :
   Level
             Normal
                                 Basic
# 90%
      (48.62, 51.76) (48.44, 51.64)
# Level
           Percentile
                                 BCa
# 90%
       (48.50, 51.70)
                        (48.88, 52.02)
# Calculations and Intervals on Original Scale
# Some BCa intervals may be unstable
# Shah's lower 90% BCa confidence interval:
# 48.82488
```

print.mimcr

Print a summary of MIMCR estimation

## **Description**

This is a method for the function print() for objects of class 'mimcr'.

# Usage

```
## S3 method for class 'mimcr'
print(x, ...)
```

# Arguments

x An object of class 'mimcr' returned by the mimcr() function.

Further arguments passed to or from other methods or arguments that can be passed down to the formatC() function.

#### **Details**

The most relevant information in an 'mimcr' object is printed.

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#### Value

The 'mimcr' object passed to the x parameter is returned invisibly.

#### See Also

```
mimcr, formatC, methods.
```

```
\# Dissolution data of one reference batch and one test batch of n = 6
# tablets each:
str(dip1)
# 'data.frame': 12 obs. of 10 variables:
# $ type : Factor w/ 2 levels "R", "T": 1 1 1 1 1 1 2 2 2 2 ...
# $ tablet: Factor w/ 6 levels "1", "2", "3", "4", ...: 1 2 3 4 5 6 1 2 3 4 ...
# $ t.5 : num 42.1 44.2 45.6 48.5 50.5 ...
# $ t.10 : num 59.9 60.2 55.8 60.4 61.8 ...
# $ t.15 : num 65.6 67.2 65.6 66.5 69.1 ...
# $ t.20 : num 71.8 70.8 70.5 73.1 72.8 ...
# $ t.30 : num 77.8 76.1 76.9 78.5 79 ...
# $ t.60 : num 85.7 83.3 83.9 85 86.9 ...
# $ t.90 : num 93.1 88 86.8 88 89.7 ...
# $ t.120 : num 94.2 89.6 90.1 93.4 90.8 ...
# Assessment of data by aid of the mimcr() function
res1 <- mimcr(data = dip1, tcol = 3:10, grouping = "type")
# Print of a summary of the assessment
print(res1)
# Results of Model-Independent Multivariate Confidence Region (MIMCR)
# approach to assess equivalence of highly variable in-vitro
# dissolution profiles of two drug product formulations
# Did the Newton-Raphson search converge? Yes
# Parameters (general):
                                       0.05
# Significance level:
# Degrees of freedom (1):
                                      7
# Degrees of freedom (2):
# Mahalanobis distance (MD):
                                      25.72
# (F) scaling factor K:
                                       0.1714
# (MD) scaling factor k:
# Hotelling's T2:
                                       1984
# Parameters specific for Tsong (1996) approach:
# Maximum tolerable average difference: 10
# Similarity limit:
                                       11.33
# Observed upper limit:
                                       31.68
# Parameters specific for Hoffelder (2016) approach:
```

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```
# Noncentrality parameter:
                                       385
# Critial F (Hoffelder):
                                       23.16
# Probability p (Hoffelder):
                                       0.7402
# Conclusions:
       Tsong (1996): Dissimilar
   Hoffelder (2016): Dissimilar
# Taking only the 15 and 90 minutes testing points into account produces a
# warning because profiles should comprise a minimum of three testing points.
res2 <- mimcr(data = dip1, tcol = c(5, 9), grouping = "type", mtad = 15,
             signif = 0.1)
print(res2)
# Warning:
# In mimcr(data = dip1, tcol = c(5, 9), grouping = "type", mtad = 15, :
# The profiles should comprise a minimum of 3 time points. The actual profiles
# comprise 2 points only.
# Results of Model-Independent Multivariate Confidence Region (MIMCR)
# approach to assess equivalence of highly variable in-vitro
# dissolution profiles of two drug product formulations
# Did the Newton-Raphson search converge? Yes
# Parameters (general):
# Significance level:
                                       0.1
# Degrees of freedom (1):
                                       2
# Degrees of freedom (2):
# Mahalanobis distance (MD):
                                      10.44
# (F) scaling factor K:
                                       1.35
# (MD) scaling factor k:
                                       3
# Hotelling's T2:
                                       327
# Parameters specific for Tsong (1996) approach:
# Maximum tolerable average difference: 15
# Similarity limit:
                                       9.631
# Observed upper limit:
                                       11.93
# Parameters specific for Hoffelder (2016) approach:
# Noncentrality parameter:
# Critial F (Hoffelder):
                                       83.57
# Probability p (Hoffelder):
                                      0.4823
# Conclusions:
       Tsong (1996): Dissimilar
   Hoffelder (2016): Dissimilar
\# Dissolution data of one reference batch and one test batch of n = 12
# capsules each:
str(dip3)
# 'data.frame': 24 obs. of 6 variables:
```

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```
# $ cap : Factor w/ 12 levels "1","2","3","4",..: 1 2 3 4 5 6 7 8 9 10 ...
# $ batch: Factor w/ 2 levels "blue", "white": 2 2 2 2 2 2 2 2 2 2 ...
# $ type : Factor w/ 2 levels "ref", "test": 1 1 1 1 1 1 1 1 1 1 ...
# $ x.15 : num 49 15 56 57 6 62 23 11 9 42 ...
# $ x.20 : num 86 59 84 87 58 90 71 64 61 81 ...
# $ x.25 : num 98 96 96 99 90 97 97 92 88 96 ...
# A successful comparison:
res3 <- mimcr(data = dip3, tcol = 4:6, grouping = "batch")
print(res3)
# Results of Model-Independent Multivariate Confidence Region (MIMCR)
# approach to assess equivalence of highly variable in-vitro
# dissolution profiles of two drug product formulations
# Did the Newton-Raphson search converge? Yes
# Parameters (general):
# Significance level:
                                       0.05
# Degrees of freedom (1):
                                       3
# Degrees of freedom (2):
                                       20
# Mahalanobis distance (MD):
                                       0.2384
# (F) scaling factor K:
                                       1.818
# (MD) scaling factor k:
                                       6
# Hotelling's T2:
                                       0.341
# Parameters specific for Tsong (1996) approach:
# Maximum tolerable average difference: 10
# Similarity limit:
                                       2.248
# Observed upper limit:
                                       1.544
# Parameters specific for Hoffelder (2016) approach:
# Noncentrality parameter:
                                     30.32
# Critial F (Hoffelder):
                                      4.899
# Probability p (Hoffelder):
                                      2.891e-08
# Conclusions:
       Tsong (1996): Similar
#
   Hoffelder (2016): Similar
```

print.mztia

Print a summary of MZTIA estimation

#### Description

This is a method for the function print() for objects of class 'mztia'.

#### Usage

```
## S3 method for class 'mztia'
print(x, ...)
```

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## Arguments

x An object of class 'mztia' returned by the mztia() function.

Further arguments passed to or from other methods or arguments that can be passed down to the print.data.frame() function.

#### **Details**

The "limits" subset (see column "frame") of the data frame that is contained in the "Data" element of the 'mztia' object is printed.

#### Value

The 'mztia' object passed to the x parameter is returned invisibly.

## See Also

```
mztia, print.data.frame, methods.
```

```
\# Dissolution data of one reference batch and one test batch of n = 6
# tablets each:
str(dip1)
# 'data.frame': 12 obs. of 10 variables:
# $ type : Factor w/ 2 levels "R", "T": 1 1 1 1 1 1 2 2 2 2 ...
# $ tablet: Factor w/ 6 levels "1","2","3","4",..: 1 2 3 4 5 6 1 2 3 4 ...
# $ t.5 : num 42.1 44.2 45.6 48.5 50.5 ...
# $ t.10 : num 59.9 60.2 55.8 60.4 61.8 ...
# $ t.15 : num 65.6 67.2 65.6 66.5 69.1 ...
# $ t.20 : num 71.8 70.8 70.5 73.1 72.8 ...
# $ t.30 : num 77.8 76.1 76.9 78.5 79 ...
# $ t.60 : num 85.7 83.3 83.9 85 86.9 ...
# $ t.90 : num 93.1 88 86.8 88 89.7 ...
# $ t.120 : num 94.2 89.6 90.1 93.4 90.8 ...
# Assessment of data (in wide format) by aid of the mztia() function
res1 <- mztia(data = dip1, shape = "wide", tcol = 3:10, grouping = "type",
             reference = "R", cap = FALSE)
# Print of a summary of the assessment
print(res1)
# Results of Martinez & Zhao Tolerance Interval (TI) Approach
# (TI limits calculated at each time point of the dissolution profiles of a set
# of reference batches)
# Time
                    LTL
                              UTL S1.LTL
                                              S1.UTL S2.LTL
          Mean
                                                                 S2.UTL
     5 46.77167 27.22641 66.31693 22.22641 71.31693 12.22641 81.31693
    10 60.13333 46.15483 74.11184 41.15483 79.11184 31.15483 89.11184
# 3 15 67.27500 56.90417 77.64583 51.90417 82.64583 41.90417 92.64583
```

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```
20 71.98667 65.44354 78.52979 60.44354 83.52979 50.44354 93.52979
     30 78.07000 69.54259 86.59741 64.54259 91.59741 54.54259 101.59741
     60 84.81667 77.20275 92.43058 72.20275 97.43058 62.20275 107.43058
     90 89.09333 76.24588 101.94079 71.24588 106.94079 61.24588 116.94079
# 8 120 91.43833 80.29321 102.58346 75.29321 107.58346 65.29321 117.58346
# Abbreviations:
   TL: Tolerance Interval Limit (TL); LTL: lower TL; UTL: upper TL;
   S1: level 1 boundary (LTL - 5) or (UTL + 5); S2: level 2 boundary
   (LTL - 15) or (UTL + 15).
# Fluid weights of 100 drink cans were measured in ounces:
str(dip5)
# 'data.frame': 100 obs. of 3 variables:
# $ type : Factor w/ 1 level "reference": 1 1 1 1 1 1 1 1 1 1 ...
# $ batch : Factor w/ 100 levels "b1", "b10", "b100",...: 1 13 24 35 46 57 68 ...
# $ weight: num 12.1 12 12 12 12 ...
# Assessment of data (in long format) by aid of the mztia() function
res2 <- mztia(data = dip5, shape = "long", tcol = 3, grouping = "type",</pre>
             reference = "reference", response = "weight", cap = FALSE,
             QS = c(5, 15) / 100)
# Print of a summary of the assessment
print(res2)
# Results of Martinez & Zhao Tolerance Interval (TI) Approach
# (TI limits calculated at each time point of the dissolution profiles of a set
# of reference batches)
# Time
         Mean
                   LTL
                            UTL
                                 S1.LTL S1.UTL S2.LTL
                                                             S2.UTL
      1 12.0093 11.87152 12.14708 11.82152 12.19708 11.72152 12.29708
# Abbreviations:
   TL: Tolerance Interval Limit (TL); LTL: lower TL; UTL: upper TL;
   S1: level 1 boundary (LTL - 0.05) or (UTL + 0.05); S2: level 2 boundary
   (LTL - 0.15) or (UTL + 0.15).
```

print.plot\_mztia

Print a plot of MZTIA estimation

## **Description**

This is a method for the function print() for objects of class 'plot\_mztia'.

#### Usage

```
## S3 method for class 'plot_mztia'
print(x, ...)
```

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## Arguments

x An object of class 'plot\_mztia' returned by the plot\_mztia() function.

Further arguments passed to or from other methods or arguments that can be passed down to the plot.boot() function.

#### Details

The element Graph of the 'plot\_mztia' object that is returned by the function plot\_mztia() is an object of class 'ggplot', generated by the function ggplot() from the 'ggplot2' package. Thus, the corresponding plot method is used for plotting. Arguments to the ggplot() function can be passed via the ... parameter.

#### Value

The 'plot\_mztia' object passed to the x parameter is returned invisibly.

#### See Also

```
mztia, plot_mztia, ggplot(), methods.
```

```
\# Dissolution data of one reference batch and one test batch of n = 6
# tablets each:
str(dip1)
# 'data.frame': 12 obs. of 10 variables:
# $ type : Factor w/ 2 levels "R","T": 1 1 1 1 1 1 2 2 2 2 ...
# $ tablet: Factor w/ 6 levels "1","2","3","4",...: 1 2 3 4 5 6 1 2 3 4 ...
# $ t.5 : num 42.1 44.2 45.6 48.5 50.5 ...
# $ t.10 : num 59.9 60.2 55.8 60.4 61.8 ...
# $ t.15 : num 65.6 67.2 65.6 66.5 69.1 ...
# $ t.20 : num 71.8 70.8 70.5 73.1 72.8 ...
# $ t.30 : num 77.8 76.1 76.9 78.5 79 ...
# $ t.60 : num 85.7 83.3 83.9 85 86.9 ...
# $ t.90 : num 93.1 88 86.8 88 89.7 ...
# $ t.120 : num 94.2 89.6 90.1 93.4 90.8 ...
# Assessment of data by aid of the mztia() function
res1 <- mztia(data = dip1, shape = "wide", tcol = 3:10, grouping = "type",
              reference = "R", cap = FALSE)
# The 'mztia' object can be passed on to the plot_mztia() function. This
# function does not produce any output but returns a 'plot_mztia' object.
gg1 <- plot_mztia(res1)</pre>
gg2 <- print(gg1)</pre>
# The print() function returns the 'plot_mztia' object invisibly.
class(gg1)
class(gg2)
```

summary.bootstrap\_f2

summary.bootstrap\_f2 Summary of the bootstrap f2 simulation

## **Description**

This is a method for the function summary() for objects of class 'bootstrap\_f2'.

# Usage

```
## S3 method for class 'bootstrap_f2'
summary(object, ...)
```

## **Arguments**

object An object of class 'bootstrap\_f2' returned by the bootstrap\_f2() function.

Further arguments passed to or from other methods or arguments that can be passed down to the print.boot() and print.bootci() functions.

#### **Details**

The elements Boot and CI of the 'bootstrap\_f2' object that is returned by the function bootstrap\_f2() are objects of type 'boot' and 'bootci', respectively, generated by the functions boot() and boot.ci(), respectively, from the 'boot' package. Thus, the corresponding print methods are used. Arguments to the print.boot() and print.bootci() functions can be passed via the ... parameter.

#### Value

The 'bootstrap\_f2' object passed to the object parameter is returned invisibly.

#### See Also

bootstrap\_f2, boot, boot.ci, print.boot, print.bootci, methods.

```
# Dissolution data of one reference batch and five test batches of n = 12
# tablets each:
str(dip2)

# 'data.frame': 72 obs. of 8 variables:
# $ type : Factor w/ 2 levels "Reference", "Test": 1 1 1 1 1 1 1 1 1 1 1 1 1 1 ...
# $ tablet: Factor w/ 12 levels "1", "2", "3", "4", ...: 1 2 3 4 5 6 7 8 9 10 ...
# $ batch : Factor w/ 6 levels "b0", "b1", "b2", ...: 1 1 1 1 1 1 1 1 1 1 1 1 ...
# $ t.0 : int 0 0 0 0 0 0 0 0 0 0 ...
# $ t.30 : num 36.1 33 35.7 32.1 36.1 34.1 32.4 39.6 34.5 38 ...
# $ t.60 : num 58.6 59.5 62.3 62.3 53.6 63.2 61.3 61.8 58 59.2 ...
# $ t.90 : num 80 80.8 83 81.3 72.6 83 80 80.4 76.9 79.3 ...
```

```
# $ t.180 : num 93.3 95.7 97.1 92.8 88.8 97.4 96.8 98.6 93.3 94 ...
# Bootstrap assessment of data (two groups) by aid of bootstrap_f2() function
# by using 'rand_mode = "complete"' (the default, randomisation of complete
# profiles)
bs1 <- bootstrap_f2(data = dip2[dip2$batch %in% c("b0", "b4"), ],
                    tcol = 5:8, grouping = "batch", rand_mode = "complete",
                   R = 200, new\_seed = 421, use\_EMA = "no")
# Summary of the assessment
summary(bs1)
# STRATIFIED BOOTSTRAP
#
# Call:
  boot(data = data, statistic = get_f2, R = R, strata = data[, grouping],
        grouping = grouping, tcol = tcol[ok])
#
# Bootstrap Statistics :
# original
               bias std. error
# t1* 50.07187 -0.02553234 0.9488015
# BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
# Based on 200 bootstrap replicates
# CALL :
   boot.ci(boot.out = t_boot, conf = confid, type = "all", L = jack$loo.values)
# Intervals :
# Level
              Normal
                                  Basic
# 90% (48.54, 51.66) (48.46, 51.71)
# Level
           Percentile
                                 BCa
# 90% (48.43, 51.68) (48.69, 51.99)
# Calculations and Intervals on Original Scale
# Some BCa intervals may be unstable
# Shah's lower 90% BCa confidence interval:
# 48.64613
# Use of 'rand_mode = "individual"' (randomisation per time point)
bs2 <- bootstrap_f2(data = dip2[dip2$batch %in% c("b0", "b4"), ],
                   tcol = 5:8, grouping = "batch", rand_mode = "individual",
                    R = 200, new\_seed = 421, use\_EMA = "no")
# Summary of the assessment
summary(bs2)
# PARAMETRIC BOOTSTRAP
```

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```
#
#
# Call:
   boot(data = data, statistic = get_f2, R = R, sim = "parametric",
        ran.gen = rand_indiv_points, mle = mle, grouping = grouping,
#
        tcol = tcol[ok], ins = seq_along(b1))
# Bootstrap Statistics :
                      std. error
   original
             bias
# t1* 50.07187 -0.1215656 0.9535517
# BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
# Based on 200 bootstrap replicates
#
# CALL :
   boot.ci(boot.out = t\_boot, conf = confid, type = "all", L = jack$loo.values)
#
# Intervals :
   Level
              Normal
                                  Basic
      (48.62, 51.76) (48.44, 51.64)
           Percentile
# Level
                                 BCa
# 90% (48.50, 51.70)
                        (48.88, 52.02)
# Calculations and Intervals on Original Scale
# Some BCa intervals may be unstable
#
# Shah's lower 90% BCa confidence interval:
# 48.82488
```

summary.mimcr

Summary of MIMCR estimation

# **Description**

This is a method for the function summary() for objects of class 'mimcr'.

# Usage

```
## S3 method for class 'mimcr'
summary(object, ...)
```

## Arguments

object An object of class 'mimcr' returned by the mimcr() function.

Further arguments passed to or from other methods or arguments that can be passed down to the formatC() function.

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#### **Details**

The most relevant information in an 'mimcr' object is printed.

#### Value

The 'mimcr' object passed to the object parameter is returned invisibly.

#### See Also

```
mimcr, formatC, methods.
```

```
\# Dissolution data of one reference batch and one test batch of n = 6
# tablets each:
str(dip1)
# 'data.frame': 12 obs. of 10 variables:
# $ type : Factor w/ 2 levels "R", "T": 1 1 1 1 1 1 2 2 2 2 ...
# $ tablet: Factor w/ 6 levels "1", "2", "3", "4", ...: 1 2 3 4 5 6 1 2 3 4 ...
# $ t.5 : num 42.1 44.2 45.6 48.5 50.5 ...
# $ t.10 : num 59.9 60.2 55.8 60.4 61.8 ...
# $ t.15 : num 65.6 67.2 65.6 66.5 69.1 ...
# $ t.20 : num 71.8 70.8 70.5 73.1 72.8 ...
# $ t.30 : num 77.8 76.1 76.9 78.5 79 ...
# $ t.60 : num 85.7 83.3 83.9 85 86.9 ...
# $ t.90 : num 93.1 88 86.8 88 89.7 ...
# $ t.120 : num 94.2 89.6 90.1 93.4 90.8 ...
# Assessment of data by aid of the mimcr() function
res1 <- mimcr(data = dip1, tcol = 3:10, grouping = "type")
# Summary of the assessment
summary(res1)
# Results of Model-Independent Multivariate Confidence Region (MIMCR)
# approach to assess equivalence of highly variable in-vitro
# dissolution profiles of two drug product formulations
# Did the Newton-Raphson search converge? Yes
# Parameters (general):
# Significance level:
                                       0.05
# Degrees of freedom (1):
                                       7
# Degrees of freedom (2):
                                       4
# Mahalanobis distance (MD):
                                       25.72
# (F) scaling factor K:
                                       0.1714
# (MD) scaling factor k:
# Hotelling's T2:
                                       1984
# Parameters specific for Tsong (1996) approach:
# Maximum tolerable average difference: 10
```

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```
# Similarity limit:
                                      11.33
# Observed upper limit:
                                      31.68
# Parameters specific for Hoffelder (2016) approach:
# Noncentrality parameter: 385
# Critial F (Hoffelder):
                                     23.16
# Probability p (Hoffelder):
                                     0.7402
# Conclusions:
       Tsong (1996): Dissimilar
   Hoffelder (2016): Dissimilar
#
# Taking only the 15 and 90 minutes testing points into account produces a
# warning because profiles should comprise a minimum of three testing points.
res2 <- mimcr(data = dip1, tcol = c(5, 9), grouping = "type", mtad = 15,
             signif = 0.1)
summary(res2)
# Warning:
# In mimcr(data = dip1, tcol = c(5, 9), grouping = "type", mtad = 15, :
# The profiles should comprise a minimum of 3 time points. The actual profiles
# comprise 2 points only.
# Results of Model-Independent Multivariate Confidence Region (MIMCR)
# approach to assess equivalence of highly variable in-vitro
# dissolution profiles of two drug product formulations
# Did the Newton-Raphson search converge? Yes
# Parameters (general):
# Significance level:
                                      0.1
# Degrees of freedom (1):
                                      2
# Degrees of freedom (2):
# Mahalanobis distance (MD):
                                     10.44
# (F) scaling factor K:
                                     1.35
# (MD) scaling factor k:
                                      3
# Hotelling's T2:
                                      327
# Parameters specific for Tsong (1996) approach:
# Maximum tolerable average difference: 15
# Similarity limit:
                                      9.631
# Observed upper limit:
                                      11.93
# Parameters specific for Hoffelder (2016) approach:
# Noncentrality parameter: 278.3
# Critial F (Hoffelder):
                                     83.57
# Probability p (Hoffelder):
                                     0.4823
# Conclusions:
       Tsong (1996): Dissimilar
   Hoffelder (2016): Dissimilar
\# Dissolution data of one reference batch and one test batch of n = 12
```

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```
# capsules each:
str(dip3)
# 'data.frame': 24 obs. of 6 variables:
# $ cap : Factor w/ 12 levels "1","2","3","4",..: 1 2 3 4 5 6 7 8 9 10 ...
# $ batch: Factor w/ 2 levels "blue", "white": 2 2 2 2 2 2 2 2 2 2 ...
# $ type : Factor w/ 2 levels "ref", "test": 1 1 1 1 1 1 1 1 1 1 ...
# $ x.15 : num 49 15 56 57 6 62 23 11 9 42 ...
# $ x.20 : num 86 59 84 87 58 90 71 64 61 81 ...
# $ x.25 : num 98 96 96 99 90 97 97 92 88 96 ...
# A successful comparison:
res3 <- mimcr(data = dip3, tcol = 4:6, grouping = "batch")
summary(res3)
# Results of Model-Independent Multivariate Confidence Region (MIMCR)
# approach to assess equivalence of highly variable in-vitro
# dissolution profiles of two drug product formulations
# Did the Newton-Raphson search converge? Yes
# Parameters (general):
  Significance level:
                                       0.05
# Degrees of freedom (1):
                                       3
# Degrees of freedom (2):
                                       20
# Mahalanobis distance (MD):
                                       0.2384
# (F) scaling factor K:
                                       1.818
# (MD) scaling factor k:
# Hotelling's T2:
                                       0.341
# Parameters specific for Tsong (1996) approach:
# Maximum tolerable average difference: 10
# Similarity limit:
                                       2.248
# Observed upper limit:
                                       1.544
# Parameters specific for Hoffelder (2016) approach:
# Noncentrality parameter:
                                      30.32
# Critial F (Hoffelder):
                                       4.899
                                 2.891e-08
# Probability p (Hoffelder):
# Conclusions:
#
       Tsong (1996): Similar
   Hoffelder (2016): Similar
```

summary.mztia

Summary of MZTIA estimation

## **Description**

This is a method for the function summary() for objects of class 'mztia'.

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## Usage

```
## S3 method for class 'mztia'
summary(object, ...)
```

#### **Arguments**

object An object of class 'mztia' returned by the mztia() function.

... Further arguments passed to or from other methods or arguments that can be passed down to the print.data.frame() function.

## **Details**

The "limits" subset (see column "frame") of the data frame that is contained in the "Data" element of the 'mztia' object is printed.

#### Value

The 'mztia' object passed to the object parameter is returned invisibly.

#### See Also

```
mztia, print.data.frame, methods.
```

```
\# Dissolution data of one reference batch and one test batch of n = 6
# tablets each:
str(dip1)
# 'data.frame': 12 obs. of 10 variables:
# $ type : Factor w/ 2 levels "R", "T": 1 1 1 1 1 1 2 2 2 2 ...
# $ tablet: Factor w/ 6 levels "1", "2", "3", "4", ...: 1 2 3 4 5 6 1 2 3 4 ...
# $ t.5 : num 42.1 44.2 45.6 48.5 50.5 ...
# $ t.10 : num 59.9 60.2 55.8 60.4 61.8 ...
# $ t.15 : num 65.6 67.2 65.6 66.5 69.1 ...
# $ t.20 : num 71.8 70.8 70.5 73.1 72.8 ...
# $ t.30 : num 77.8 76.1 76.9 78.5 79 ...
# $ t.60 : num 85.7 83.3 83.9 85 86.9 ...
# $ t.90 : num 93.1 88 86.8 88 89.7 ...
# $ t.120 : num 94.2 89.6 90.1 93.4 90.8 ...
# Assessment of data (in wide format) by aid of the mztia() function
res1 <- mztia(data = dip1, shape = "wide", tcol = 3:10, grouping = "type",
             reference = "R", cap = FALSE)
# Summary of the assessment
summary(res1)
# Results of Martinez & Zhao Tolerance Interval (TI) Approach
# (TI limits calculated at each time point of the dissolution profiles of a set
# of reference batches)
```

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```
# Time
                    LTL
                              UTL S1.LTL
                                              S1.UTL S2.LTL
          Mean
# 1
      5 46.77167 27.22641 66.31693 22.22641 71.31693 12.22641 81.31693
     10 60.13333 46.15483 74.11184 41.15483 79.11184 31.15483 89.11184
# 3
     15 67.27500 56.90417 77.64583 51.90417 82.64583 41.90417 92.64583
     20 71.98667 65.44354 78.52979 60.44354 83.52979 50.44354 93.52979
# 5
     30 78.07000 69.54259 86.59741 64.54259 91.59741 54.54259 101.59741
     60 84.81667 77.20275 92.43058 72.20275 97.43058 62.20275 107.43058
     90 89.09333 76.24588 101.94079 71.24588 106.94079 61.24588 116.94079
# 8 120 91.43833 80.29321 102.58346 75.29321 107.58346 65.29321 117.58346
# Abbreviations:
   TL: Tolerance Interval Limit (TL); LTL: lower TL; UTL: upper TL;
   S1: level 1 boundary (LTL - 5) or (UTL + 5); S2: level 2 boundary
   (LTL - 15) or (UTL + 15).
# Fluid weights of 100 drink cans were measured in ounces:
str(dip5)
# 'data.frame': 100 obs. of 3 variables:
# $ type : Factor w/ 1 level "reference": 1 1 1 1 1 1 1 1 1 1 ...
# $ batch : Factor w/ 100 levels "b1", "b10", "b100", ...: 1 13 24 35 46 57 68 ...
# $ weight: num 12.1 12 12 12 12 ...
# Assessment of data (in long format) by aid of the mztia() function
res2 <- mztia(data = dip5, shape = "long", tcol = 3, grouping = "type"</pre>
             reference = "reference", response = "weight", cap = FALSE,
             QS = c(5, 15) / 100)
# Summary of the assessment
summary(res2)
# Results of Martinez & Zhao Tolerance Interval (TI) Approach
# (TI limits calculated at each time point of the dissolution profiles of a set
# of reference batches)
# Time
                   LTL
                            UTL S1.LTL S1.UTL S2.LTL S2.UTL
         Mean
      1 12.0093 11.87152 12.14708 11.82152 12.19708 11.72152 12.29708
# Abbreviations:
   TL: Tolerance Interval Limit (TL); LTL: lower TL; UTL: upper TL;
   S1: level 1 boundary (LTL - 0.05) or (UTL + 0.05); S2: level 2 boundary
   (LTL - 0.15) or (UTL + 0.15).
```

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