# Package 'simdd'

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Type Package

 ${\it simdd-package} \qquad {\it simdd-Simulation\ of\ Fisher\ Bingham\ and\ Related\ Directional\ Distributions}$ 

### Description

Simulation methods for the Fisher Bingham distribution on the unit sphere, the matrix Bingham distribution on a Grassmann manifold, the matrix Fisher distribution on SO(3), and the bivariate von Mises sine model on the torus. The methods use the first ever general purpose acceptance/rejection simulation algorithm for the Bingham distribution and are described fully by Kent, Ganeiber and Mardia (2018). These methods superseded earlier MCMC simulation methods and are more general than earlier simulation methods. The methods can be slower in specific situations where there are existing non-MCMC simulation methods (see Section 8 of Kent, Ganeiber and Mardia (2018) for further details).

#### Author(s)

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#### References

John T. Kent, Asaad M. Ganeiber & Kanti V. Mardia (2018) A New Unified Approach for the Simulation of a Wide Class of Directional Distributions, Journal of Computational and Graphical Statistics, 27:2, 291-301, DOI: doi:10.1080/10618600.2017.1390468

#### **Examples**

```
# simulate 20 random unit vectors from a Bingham distribution # with concentration matrix diag(c(1,2,-3)). X=rBingham(20,c(1,2,-3)) # check the efficiency of the simulation algorithm attr(X,"summary")
```

Simulate Fisher Bingham and related distributions

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## **Description**

Simulate the Fisher Bingham distribution on the unit sphere, the matrix Bingham distribution on a Grassmann manifold, the matrix Fisher distribution on SO(3), and the bivariate von Mises sin model on the torus = product of two circles.

#### Usage

```
rFisherBingham(nsim, mu = 0, Aplus = 0, q = dimset(mu, Aplus), mtop = 1000)
rBingham(nsim, Aplus, q = dimq(Aplus), mtop = 1000)
rBingham.Grassmann(nsim, Aplus = 0, q = dimq(Aplus), r = 1, mtop = 1000)
rFisher.SO3(nsim, Fmat, mtop = 1000)
rvMsin.torus(nsim, k1, k2, alpha, mtop=1000)
rBessel(nsim, k1, k2, alpha, mtop=1000)
```

#### Arguments

nsim	the desired number of simulations
mu	a vector of length q. If mu is entered as a scalar, it is interpreted as the vector $(0, \ldots, 0, mu)$ with zeros except in the final coordinate.
Aplus	a symmetric q by q matrix. If Aplus is given as a scalar, it is interpreted as the q by q matrix of zeros. If Aplus is given as a vector, it is interpreted as the matrix diag(Aplus).
q	The simulation is done on the unit sphere in $\mathbb{R}^q$ . Thus for the circle, q=2. Note that q only needs to be given explicitly if both mu and Aplus are scalars. In the default setting the internal function dimset, q=dimset(mu,Aplus), determines q from the length of mu or the dimension of Aplus, if it can.
r	For rBingham. Grassmann, r denotes the number of columns (1 $\leq$ r $\leq$ q-1) in the matrix Bingham distribution
mtop	The maximum number of attempts to generate nsim simulations. A finite mtop prevents infinite loops in extreme situations. If mtop is reached there will be a warning.
Fmat	The 3 by 3 parameter matrix for the matrix Fisher distribution in the function rFisher. S03.
k1, k2, alpha	The two concentration parameters and the interaction parameter in the function rvMsin.torus.

#### **Details**

The Fisher Bingham distribution on the unit sphere in  $\mathbb{R}^q$  has density proportional to

$$\exp(\mathbf{m}\mathbf{u}^Tx + x^T\mathbf{Aplus}x)$$

where x is a unit vector in  $\mathbb{R}^q$ , and mu (q-dimensional vector) and Aplus (q by q symmetric) are parameters.

The matrix Bingham distribution on q by r matrices X whose columns are orthonormal, is given by the density proportional to

$$\exp(trace(X^T \mathrm{Aplus} X)).$$

The Bingham distribution on the unit sphere in  $\mathbb{R}^q$  can be simulated using (a) rBingham, (b) rFisherBingham with mu=0, and (c) rBingham. Grassmann with r=1. Choice (a) will be fastest.

The Fisher distribution can be simulated using rFisherBingham with Aplus=0.

The matrix Fisher distribution on SO(3) has density proportional to

$$\exp(trace(\mathsf{Fmat}^T X))$$

where X is a 3 by 3 rotation matrix, and Fmat is a 3 by 3 parameter matrix.

The bivariate von Mises sine model on the torus has density proportional to

$$\exp(k1\cos(\theta) + k2\cos(\phi) + alpha\sin(\theta)\sin(\phi))$$

for two angles  $\theta$ ,  $\phi$ . The Bessel density is obtained from the bivariate von Mises sine model as the marginal density of  $\theta$ .

If mtop is reached before obtaining nsim simulations then a warning is created and the returned array will have fewer than nsim rows.

#### Value

For rBingham and rFisherBingham, the output is an nsim by q matrix. Each row is a simulated unit vector.

For rBingham. Grassmann, the output is an nsim by q by r array. For each value of the first index, the result is a simulated q by r matrix with orthonormal columns.

For rFisher. S03, the output is an nsim by r by r array. For each value of the first index, the result is a simulated r by r matrix rotation matrix.

For rvMsin. torus, the output is an nsim by 4 matrix, with each row containing the simulated value of  $(\cos(\theta), \sin(\theta), \cos(\phi), \sin(\phi))$ .

For rBessel, the output is an nsim by 2 matrix containing the marginal simulated values of  $(\cos(\theta), \sin(\theta))$ .

In all cases, the output has an attribute summary, which is a vector of length 6 summarizing some details about the number of simulations needed in the acceptance/rejection algorithm. The key element of this vector is called efficiency, a number between 0 and 1, where 1 means that all the simulated values from the envelope distribution have been accepted.

- ntry is the number of simulations drawn from the envelope distribution.
- efficiency is the proportion of simulations drawn from the envelope distribution that were accepted.
- success is 1 when simulations were completed, and 0 otherwise. Usually the simulations are incomplete because the number of iterations (in entry mloops) has reached the maximum mtop.
- mloops is the number of iterations used.
- minfg is the smallest observed value of the envelope.
- maxfg is the largest observed value of the envelope.

#### Author(s)

J T Kent

#### References

John T. Kent, Asaad M. Ganeiber & Kanti V. Mardia (2018) A New Unified Approach for the Simulation of a Wide Class of Directional Distributions, Journal of Computational and Graphical Statistics, 27:2, 291-301, DOI: doi:10.1080/10618600.2017.1390468

#### See Also

The function rvm in the CRAN library CircStats provides a more efficient method to simulate from the von Mises distribution (i.e. the Fisher distribution with q=2) than rFisherBingham.

### **Examples**

```
X1=rBingham(10,c(1,2,-3))
attr(X1,"summary")
X2=rFisherBingham(10,c(3,2,1),c(1,2,-3))
X3=rBingham.Grassmann(10,c(1,2,-3),r=2)
X4=rFisher.SO3(10,matrix(c(1,2,3,4,2,1,0,1,2),3,3))
X5=rvMsin.torus(10,2.2,3.1,1.3)
X6=rBessel(10,2.2,3.1,1.3)
```

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