Abstract

This paper applies mortality projection techniques (Lee Carter) to the evaluation of retirement costs. The main purpose of the paper is to show how R can be successfully used to perform life expectancy projections with practical actuarial applications for annuity insurances and social security issues. **demography** and **lifecontingencies** packages will be used. The analysis performed within this paper are mechanical and intended for didactic purposes.

Mortality projection with **demography** and **lifecontingencies** packages

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1 Introduction

Mortality across any ages is showing continuous reduction across the world. Moreover, a reliable projection of mortality is very important to accurately estimate the cost of life - insurance and pension annuities.
Methodologies have been developed to project the evolution of the mortality. Lee - Carter method is probably the most well known, among these.
The Lee Cartel forecasts results could be used to project a life table for each specific cohort (year) of birth on which pension annuities projection could be fit.

Our exercise will be based on Italian data downloaded from the Human Mortality Databases via **demography** package dedicated function. We will use **demography** and **forecast** package sto fit Lee - Carter model and perform 100 - years in advance extrapolations.
Finally, **lifecontingencies** package will be used to project the cost of a pension annuity, $\ddot{a}_x^{(m)}$ for the cohorts of 1940, 1950, ..., 2000 borns cohorts. Following demographic and economic assumptions will be hold:

- x, the retirement age will be set equal to 65 regardless the cohort.
- m, the number of fractional payments per year, will be equal to 12.
- $\ddot{a}_x^{(m)}$ to be the actuarial present value of a yearly annuity of 1 monetary unit. The annuity will be evaluated assuming an interest rate of 4% and an inflation rate of 2%.

The projection has been performed using a mechanical approach, since the purpose of this paper lies in showing the procedure instead of providing sensible results.
Most of this paper is based on the examples provided in [Charpentier, 2012] and [Charpentier and Dutang, 2013] online manual.

2 Fitting Lee Carter model

Lee Carter original model, [Lee and Carter, 1992], assumes that the mortality (hazard rate) for age x in calendar year t can be expressed as Equation 1.

$$\ln \mu_{x,t} = a_x + b_x k_t + \varepsilon_{x,t} \tag{1}$$

Term in Equation 1 can be interpreted as follows:

1. a_x to be a base mortality rate for age x.
2. k_t to be the calendar year - related latent variable factor.
3. b_x is the sensitivity of age x to factor k_t.
4. $\varepsilon_{x,t}$ represents the residual component.

The one - year survival probability at age x during calendar year t is expressed by Equation 2. Equation 2 assumes constant force of mortality to hold between $[x, x+t)$ and that $\mu_x \sim m_x$, that is the force of mortality to be approximated by the central rate of mortality [1].

$$p_{x,t} = \exp\left(-\mu_{x,t}\right) \sim \exp\left(-m_{x,t}\right) \tag{2}$$

A longitudinal life table for the cohort of born in calendar year YYYY can be created selecting all $p_{x,t}$ for which $t - x = YYYY$. We will perform such exercise on Italy HMD data, saved in mortalityDataset db.

```
R> library(demography)
R> library(forecast)
R> library(lifecontingencies)
R> load(file="mortalityDatasets.RData")
```

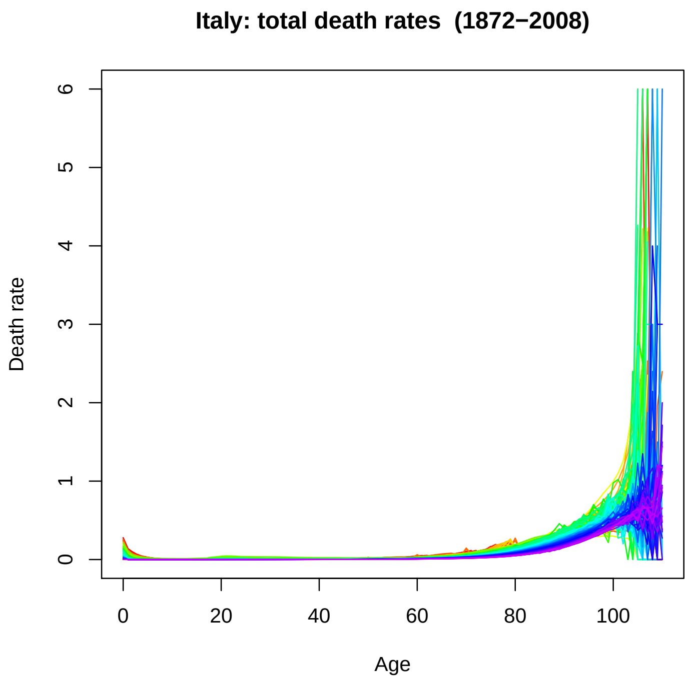
Following code creates a **demogdata** object from HDM data structure.

```
R> italyDemo<-demogdata(data=italyMx$rate$total,
+                      pop=italyMx$pop$total,
+                      name="total",
+                      ages=italyMx$age,
+                      years=italyMx$year,
+                      type="mortality",
+                      label="Italy",lambda=1)
```

Plot method is available on **demogdata**.

```
R>          plot(italyDemo)
```

[1] As shown in [Wright,], if $p_{x,t}$ were assumed linear between the two consecutive integer ages, we could write $m_x = \frac{q_x}{1-\frac{1}{2}q_x}$

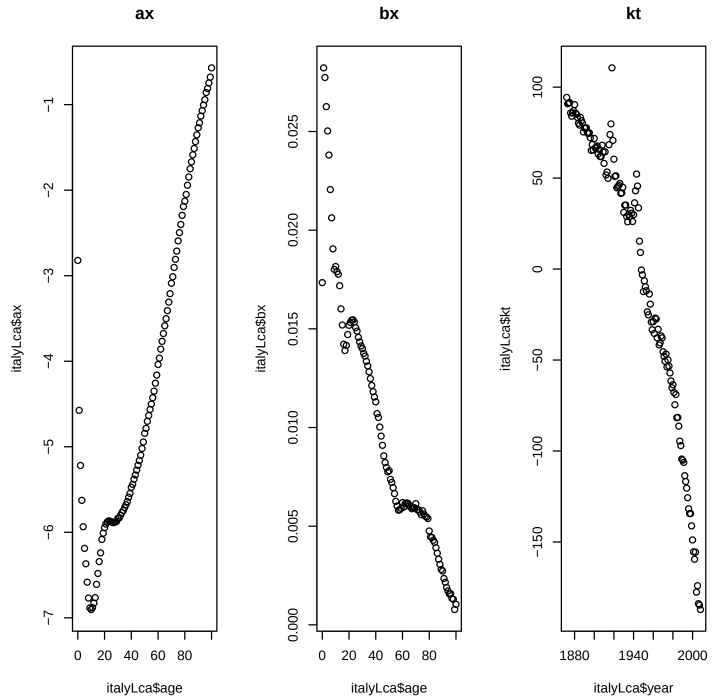


To fit Lee - Carter model (without going throught logaritms) we do

```
R> italyLca<-lca(italyDemo)
```

lca returned object allows us to inspect a_x, b_x and k_t.

```
R>          par(mfrow=c(1,3))
R>          plot(x=italyLca$age, y=italyLca$ax, main="ax")
R>          plot(x=italyLca$age, y=italyLca$bx, main="bx")
R>          plot(x=italyLca$year, y=italyLca$kt, main="kt")
```

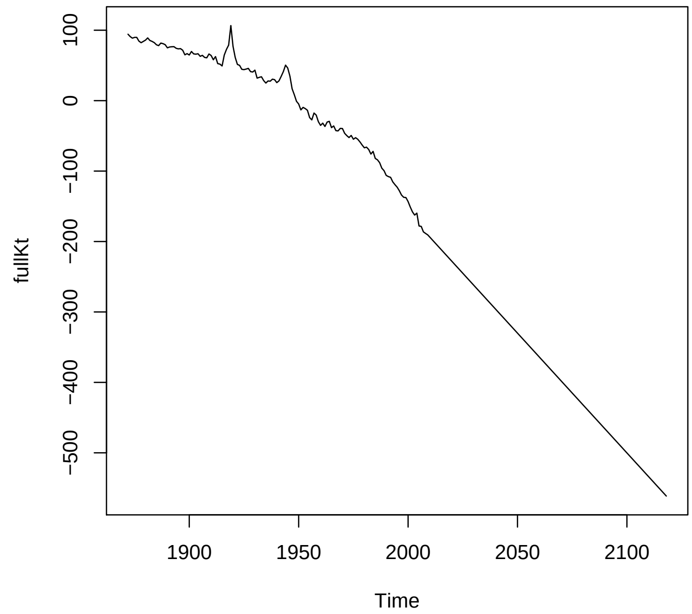


We can therefore use **forecast** package to project the future k_ts (up to 110).

```
R> ktSeries<-italyLca$kt
R> ktArima<-auto.arima(ktSeries,allowdrift=TRUE,max.order=20)
R> ktArimaForecasts<-forecast(ktArima, h=110)
R> fullKt<-ts(c(ktArimaForecasts$fitted, ktArimaForecasts$mean),start=1872)
```

and project results

```
R> plot(fullKt)
```



3 Perform actuarial projections

Then we create a function to project life table depending by year of birth, using results from Lee - Carter model. In particular, for ages $0, 1, \ldots, \tau$ on which Lee-Carter model has been fit Equation 3 apply, while for extreme ages, $\tau+1, \ldots, \omega$ on which no data were provided, it has been assumed that on year probability decreases evenly in 20 steps.

$$\begin{aligned} \ln \hat{\mu_{x,t}} &= a_x + b_x k_t \\ \hat{p}_{x,t} &= \exp\left(-\hat{\mu}_{x,t}\right) \end{aligned} \tag{3}$$

```
R> createActuarialTable<-function(yearOfBirth){
+         #get projected Px
+         ktSubset<-window(fullKt, start=yearOfBirth)
+         predictionTable<-data.frame(age=italyLca$age,ax=italyLca$ax,bx=italyLca$bx)
+         predictionTable$kt=ktSubset[1:nrow(predictionTable)]
+         predictionTable$mux=with(predictionTable,exp(ax+bx*kt)) #fit mux
+         predictionTable$px=with(predictionTable,exp(-mux)) #get px
+         fittedPx=predictionTable$px #add px to table
+         px4Completion=seq(from=predictionTable$px[length(fittedPx)], to=0, length=20)
+         totalPx=c(fittedPx,px4Completion[2:length(px4Completion)])
+         #create life table
+         irate=1.04/1.02-1
+
+         cohortLt=probs2lifetable(probs=totalPx, radix=100000,type="px", name=paste("Coh
```

```
+          cohortAct=new("actuarialtable",x=cohortLt@x, lx=cohortLt@lx,
+                          interest=irate, name=cohortLt@name)
+          return(cohortAct)
+          }
R>
```

We can therefore calculate the APV of $\ddot{a}_{65}^{(12)}$ for the selected cohorts.

```
R>         getAnnuityAPV<-function(yearOfBirth) {
+                  actuarialTable<-createActuarialTable(yearOfBirth)
+                  out=axn(actuarialTable,x=65,m=12)
+                  return(out)
+          }
R>         for(i in seq(1920,2000,by=10)) {
+                  cat("For cohort ",i, " the expected lifetime at birth is",
+                                  round(exn(createActuarialTable(i)),2),
+                                  " and the APV is :",round(getAnnuityAPV(i),2),"\n")
+
+          }

For cohort  1920  the expected lifetime at birth is 51.75  and the APV is : 5.45
For cohort  1930  the expected lifetime at birth is 63.15  and the APV is : 6.35
For cohort  1940  the expected lifetime at birth is 67.77  and the APV is : 7.09
For cohort  1950  the expected lifetime at birth is 76.19  and the APV is : 7.71
For cohort  1960  the expected lifetime at birth is 80.6  and the APV is : 8.29
For cohort  1970  the expected lifetime at birth is 82.93  and the APV is : 8.84
For cohort  1980  the expected lifetime at birth is 85.68  and the APV is : 9.35
For cohort  1990  the expected lifetime at birth is 87.96  and the APV is : 9.82
For cohort  2000  the expected lifetime at birth is 89.51  and the APV is : 10.27
```

References

[Charpentier, 2012] Charpentier, A. (2012). Actuarial Science with R 2: life insurance and mortality tables. http://freakonometrics.blog.free.fr/index.php?post/2012/04/04/Life-insurance,-with-R,-Meielisalp. Accessed: 04/11/2012.

[Charpentier and Dutang, 2013] Charpentier, A. and Dutang, C. (2013). *Actuariat Avec R*. http://cran.r-project.org/doc/contrib/Charpentier_Dutang_actuariat_avec_R.pdf.

[Lee and Carter, 1992] Lee, R. and Carter, L. (1992). Modeling and forecasting u.s. mortality. *Journal of the American Statistical Association*, 87(419):659–675.

[Wright,] Wright, D. Expectation of life. http://www.staff.city.ac.uk/b.d.rickayzen/wright%20contingencies%20notes.pdf. Accessed: 27/01/2013.