The mbbefd Package: A Package for handling MBBEFD exposure curves in R

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Abstract

The package models MBBEFD distribution providing density, quantile, distribution and random generation functions. In addition it provides exposure curves for the MBBEFD distribution family.

Keywords: mbbefd, exposure curves, reinsurance, non-life insurance.

1. Introduction

The **mbbefd** package provides function to use Maxwell-Bolzano, Bose-Einstein, Fermi-Dirac probability distributions, introduced by (BERNEGGER 1997), within R statistical software (R Core Team 2013).
Such kind of distributions are widely used in the pricing of non-life reinsurance contracts and yet they are not present in any R package.

The paper is structured as follows: Section 2 discusses review the theory (mathematics and actuarial application) of MBBEFD distributions, Section 3 shows the package's features, applied examples are shown in Section 4 while the issue of fitting MBBEFD curves to empirical data is discussed in Section 5.

2. Exposure curve review

Within actuarial jargon, an exposure curve is a distribution that shows the ratio between the expected limited loss at various limits and the expected unlimited loss. They are usually to rate large commercial risks' exposures and non-proportional reinsurance treaties. In mathematical notation, if IV is the insured value and d the ratio of loss x to IV the exposure curve $G(d)$ is defined as Equation 1 displays.

$$G(d) = \frac{E\left[\min\left(d*IV, x\right)\right]}{E\left[x\right]} = \frac{\int_0^{d*IV}\left(1-F\left(x\right)\right)dx}{\int_0^{\infty} x*f\left(x\right)dx} = \frac{\int_0^{d*IV} S\left(x\right)dx}{\int_0^{\infty} S\left(x\right)dx} \tag{1}$$

Whilst losses normally lie in the interval $0,\ldots,\infty$ for the rest of the paper it will be assumes x to represent a normalized loss in the interval $0,\ldots,PML$, being PML the so - called maximum probable loss (in other words, the maximum loss it is thought to can happen). Therefore

x would represent a percentage loss with respect to a maximum, i.e., a destruction rate.

BERNEGGER (1997) and **?** provide a discussion on the actuarial theory regarding such curve. In particular, the curves discussed by BERNEGGER (1997) are of the form expressed by Equation 2.

$$\begin{cases} G(x) = \frac{\ln(a+b^x)-\ln(a+1)}{\ln(a+b)-\ln(a+1)} \\ x \in [0,1] \end{cases} \tag{2}$$

It can be shown that $G(0) = 0$, $G(1) = 1$, $dG(d) \geq 0$ and $ddG(d) \leq 0$. Using some calculus on Equation 2 it can be shown that the expected value is equal to $\mu = \frac{1}{dG(0)}$.

The probability of a total loss, p, is expressed by Equation 3.

$$p = 1 - F\left(1^-\right) = \frac{1}{g} = \frac{dG(1)}{dG(0)} = \frac{(a+1)*b}{a+b} \tag{3}$$

3. The MBBEFD class and its related package

```
R> library(mbbefd)
```

The `mbbefdExposure` function evaluates the exposure curve for a given destruction rate x, given either a and b, or b and g. Figure 2 displays the destruction rate by level of x, for an exposure cuve of parameters $a = 0.2$ and $b = 0.04$

4. Applied examples

The curve can be use to price property coverage and associate reinsurance treaties. Suppose a property expected loss to be 40K, MPL to be 2MLN. An XL coverage is available with a retention of 1Mln. The exposure curve that characterize the property is the usual one. Therefore the percentage of loss net and ceded is determined as it follows

```
R> net<-mbbefdExposure(x=1/2, a=0.2,b=0.04)*40000
R> ceded<-40000-net
```

and the expected loss as a percentage of total insured value is

```
R> expectedLoss<-1/dG(x=0,a=0.2,b=0.04)*40000
R> expectedLoss

[1] 24000
```

The probability of a maximum loss for such exposure curve is obtained evaluating the survival function at 1

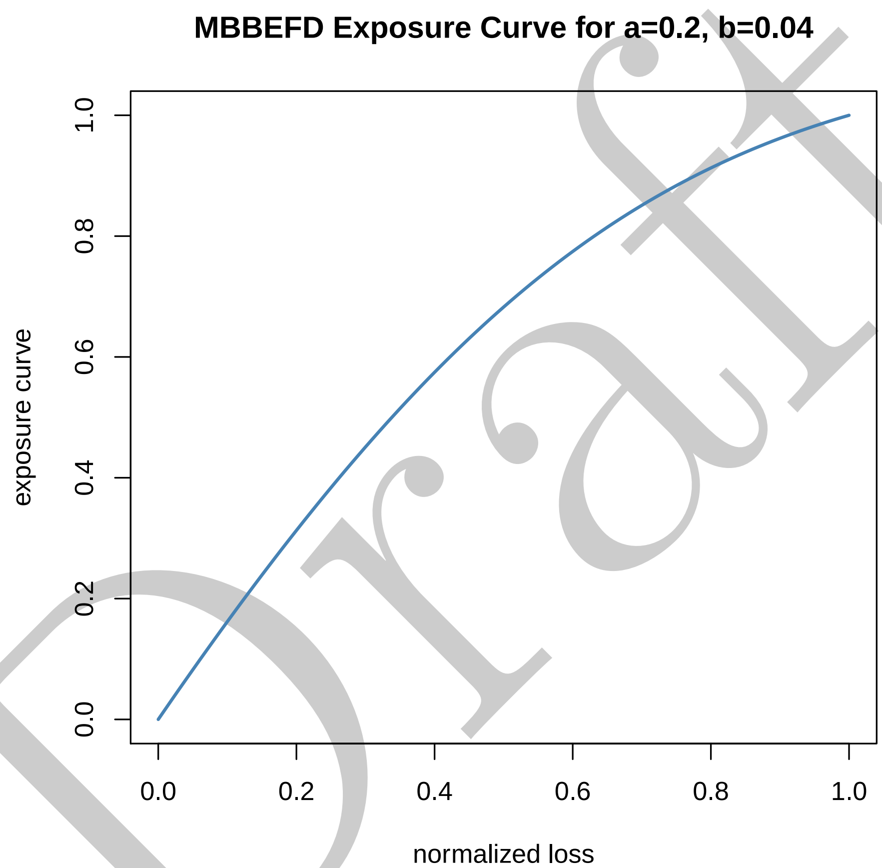

Figure 1: Exposure curve example

```
R> pTotalLoss<-1-pmbbefd(x=1,a=0.2,b=0.04)
R> pTotalLoss
```

```
[1] 0.2
```

Quantile functions, distribution functions and density functions are defined as well. For example, the 60th percentile of the distribution above defined (i.e., how bad can be in 60% of cases in terms of destruction rate) is

```
R> qmbbefd(p=0.6,a=0.2,b=0.04)
```

```
[1] 0.7153383
```

whilst a loss worse than 80% of IV could happen in

```
R> 100*(1-pmbbefd(x=0.8,a=0.2,b=0.04))
```

```
[1] 33.0895
```

cases out of 100.

It would be possible to simulate variates from the MBBEFD distribution using the random generation command `rmbbefd`.

```
R> simulatedLosses<-rmbbefd(n=10000,a=0.2,b=0.04)
R> mean(simulatedLosses)
```

```
[1] 0.597828
```

```
R> sum(simulatedLosses==1)/length(simulatedLosses)
```

```
[1] 0.1949
```

Finally another way to show the probability of total loss to be greater than zero is to show that the (numerical) integral between 0 and 1 of the density function is lower than 1, that is $1 - F(1^-)$.

```
R> integrate(dmbbefd,lower=0, upper=1, a=0.2, b=0.04)
```

```
0.8 with absolute error < 2.4e-13
```

5. Fitting MBBEFD curves

TO BE COMPLETED

References

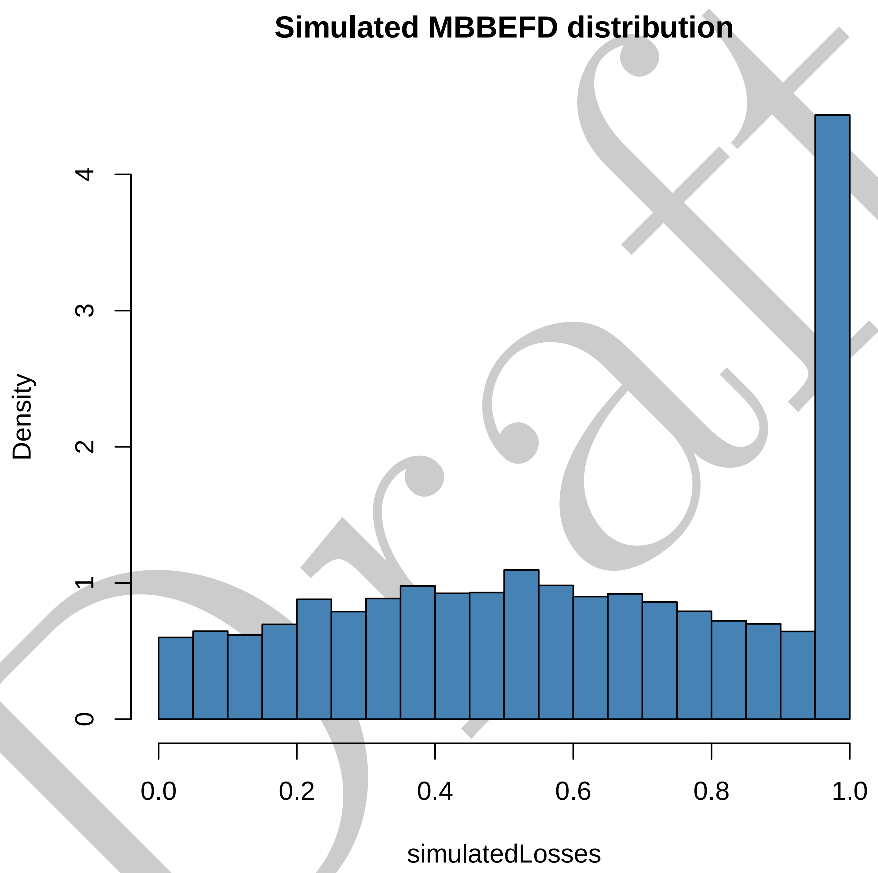

Figure 2: Exposure curve example

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