Kenneth Train's exercises using the mlogit package for R

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Abstract

This text presents the answers to the exercises provided by Kenneth Train on his web site using the **mlogit** package for R.

Keywords:~discrete choice models, maximum likelihood estimation, R, econometrics.

1. Multinomial logit model

1. The problem set uses data on choice of heating system in California houses. The data set Heating from the **mlogit** package contains the data in R format. The observations consist of single-family houses in California that were newly built and had central air-conditioning. The choice is among heating systems. Five types of systems are considered to have been possible:

1. gas central (gc),
2. gas room (gr),
3. electric central (ec),
4. electric room (er),
5. heat pump (hp).

There are 900 observations with the following variables:

- idcase gives the observation number (1-900),
- depvar identifies the chosen alternative (gc, gr, ec, er, hp),
- ic.alt is the installation cost for the 5 alternatives,
- oc.alt is the annual operating cost for the 5 alternatives,
- income is the annual income of the household,

- agehed is the age of the household head,
- rooms is the number of rooms in the house,
- region a factor with levels ncostl (northern coastal region), scostl (southern coastal region), mountn (mountain region), valley (central valley region).

Note that the attributes of the alternatives, namely, installation cost and operating cost, take a different value for each alternative. Therefore, there are 5 installation costs (one for each of the 5 systems) and 5 operating costs. To estimate the logit model, the researcher needs data on the attributes of all the alternatives, not just the attributes for the chosen alternative. For example, it is not sufficient for the researcher to determine how much was paid for the system that was actually installed (ie., the bill for the installation). The researcher needs to determine how much it would have cost to install each of the systems if they had been installed. The importance of costs in the choice process (i.e., the coefficients of installation and operating costs) is determined through comparison of the costs of the chosen system with the costs of the non-chosen systems.

For these data, the costs were calculated as the amount the system would cost if it were installed in the house, given the characteristics of the house (such as size), the price of gas and electricity in the house location, and the weather conditions in the area (which determine the necessary capacity of the system and the amount it will be run.) These cost are conditional on the house having central air-conditioning. (That's why the installation cost of gas central is lower than that for gas room: the central system can use the air-conditioning ducts that have been installed.)

In a logit model, each variable takes a different value in each alternative. So, in our case, for example, we want to know the coefficient of installation cost in the logit model of system choice. The variable installation cost in the model actually consists of five variables in the dataset: ic.gc, ic.gr, ic.ec, ic.er and ic.hp, for the installation costs of the five systems. In the current code, there are two variables in the logit model. The first variable is called ic for installation cost. This variable consists of five variables in the dataset: ic.gc in the first alternative, ic.gr in the second alternative, etc.

2. Run a model with installation cost and operating cost, without intercepts

(a) Do the estimated coefficients have the expected signs?

```
R> library("mlogit")
R> data("Heating", package = "mlogit")
R> H <- mlogit.data(Heating, shape = "wide", choice = "depvar",
+     varying = c(3:12))
R> m <- mlogit(depvar ~ ic + oc | 0, H)
R> summary(m)

Call:
mlogit(formula = depvar ~ ic + oc | 0, data = H, method = "nr",
    print.level = 0)
```

```
Frequencies of alternatives:
      ec       er       gc       gr       hp
0.071111 0.093333 0.636667 0.143333 0.055556

nr method
4 iterations, 0h:0m:0s
g'(-H)^-1g = 1.56E-07
gradient close to zero

Coefficients :
      Estimate  Std. Error t-value  Pr(>|t|)
ic -0.00623187  0.00035277 -17.665 < 2.2e-16 ***
oc -0.00458008  0.00032216 -14.217 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log-Likelihood: -1095.2
```

Yes, they are negative as expected, meaning that as the cost of a system rises (and the costs of the other systems remain the same) the probability of that system being chosen falls.

(b) Are both coefficients significantly different from zero?

Yes, the t-statistics are greater than 1.96, which is the critical level for 95% confidence level.

(c) How closely do the average probabilities match the shares of customers choosing each alternative?

```
R> apply(fitted(m, outcome = FALSE), 2, mean)
```

```
        ec         er         gc         gr         hp
0.10413057 0.05141477 0.51695653 0.24030898 0.08718915
```

Not very well. 63.67% of the sample chose gc (as shown at the top of the summary) and yet the estimated model gives an average probability of only 51.695%. The other alternatives are also fairly poorly predicted. We will find how to fix this problem in one of the models below.

(d) The ratio of coefficients usually provides economically meaningful information. The willingness to pay (*wtp*) through higher installation cost for a one-dollar reduction in operating costs is the ratio of the operating cost coefficient to the installation cost coefficient. What is the estimated *wtp* from this model? Is it reasonable in magnitude?

$$U = \beta_{ic} ic + \beta_{oc} oc$$

$$dU = \beta_{ic} dic + \beta_{oc} doc = 0 \Rightarrow -\frac{dic}{doc} \mid_{dU=0} = \frac{\beta_{oc}}{\beta_{ic}}$$

```
R> coef(m)["oc"]/coef(m)["ic"]
```

```
       oc
0.7349453
```

The model implies that the decision-maker is willing to pay \$.73 (ie., 73 cents) in higher installation cost in order to reduce annual operating costs by \$1.

A \$1 reduction in annual operating costs recurs each year. It is unreasonable to think that the decision-maker is only willing to pay only 73 cents as a one-time payment in return for a \$1/year stream of saving. This unreasonable implication is another reason (along with the inaccurate average probabilities) to believe this model is not so good. We will find below how the model can be improved.

(e) We can use the estimated wtp to obtain an estimate of the discount rate that is implied by the model of choice of operating system. The present value of the future operating costs is the discounted sum of operating costs over the life of the system: $PV = \sum_{t=1}^{L} \frac{OC}{(1+r)^t}$ where r is the discount rate and L being the life of the system. As L rises, the PV approaches OC/r. Therefore, for a system with a sufficiently long life (which we will assume these systems have), a one-dollar reduction in OC reduces the present value of future operating costs by $(1/r)$. This means that if the person choosing the system were incurring the installation costs and the operating costs over the life of the system, and rationally traded-off the two at a discount rate of r, the decisionmaker's wtp for operating cost reductions would be $(1/r)$. Given this, what value of r is implied by the estimated wtp that you calculated in part (c)? Is this reasonable?

$U = aLC$ where LC is lifecycle cost, equal to the sum of installation cost and the present value of operating costs: $LC = IC + (1/r)OC$. Substituting, we have $U = aIC + (a/r)OC$.

The models estimates a as -0.00623 and a/r as -0.00457. So $r = a/(a/r) = -.000623/.00457 = 1.36$ or 136% discount rate. This is not reasonable, because it is far too high.

3. Estimate a model that imposes the constraint that $r = 0.12$ (such that $wtp = 8.33$). Test the hypothesis that $r = 0.12$.

To impose this constraint, we create a lifecycle cost that embodies the constraint $lcc = ic + oc/0.12$ and estimate the model with this variable.

```
R> H$lcc = H$ic + H$oc/0.12
R> mlcc <- mlogit(depvar ~ lcc | 0, H)
R> lrtest(m, mlcc)

Likelihood ratio test

Model 1: depvar ~ ic + oc | 0
Model 2: depvar ~ lcc | 0
  #Df  LogLik Df  Chisq Pr(>Chisq)
1   2 -1095.2
2   1 -1248.7 -1 306.93  < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

R> qchisq(0.05, df = 1, lower.tail = FALSE)

[1] 3.841459
```

We perform a likelihood ratio test. The LL for this constrained model is -1248.7. The LL for the unconstrained model is -1095.2. The test statistic is twice the difference in LL: $2(1248.7 - 1095.2) = 307$. This test is for one restriction (ie a restiction on the relation of the coefficient of operating cost to that of installation cost.) We therefore compare 307 with the critical value of chi-squared with 1 degree of freedom. This critical value for 95% confidence is 3.8. Since the statistic exceeds the critical value, we reject the hypothesis that $r = 0.12$.

4. Add alternative-specific constants to the model. With J alternatives, at most $J - 1$ alternative-specific constants can be estimated. The coefficients of $J - 1$ constants are interpreted as relative to alternative Jth alternative. Normalize the constant for the alternative hp to 0.

(a) How well do the estimated probabilities match the shares of customers choosing each alternative?

```
R> mc <- mlogit(depvar ~ ic + oc, H, reflevel = "hp")
R> summary(mc)

Call:
mlogit(formula = depvar ~ ic + oc, data = H, reflevel = "hp",
    method = "nr", print.level = 0)

Frequencies of alternatives:
      hp       ec       er       gc       gr
0.055556 0.071111 0.093333 0.636667 0.143333

nr method
6 iterations, 0h:0m:0s
g'(-H)^-1g = 9.58E-06
successive fonction values within tolerance limits

Coefficients :
         Estimate  Std. Error t-value  Pr(>|t|)
altec  1.65884594  0.44841936  3.6993 0.0002162 ***
alter  1.85343697  0.36195509  5.1206 3.045e-07 ***
altgc  1.71097930  0.22674214  7.5459 4.485e-14 ***
altgr  0.30826328  0.20659222  1.4921 0.1356640
ic    -0.00153315  0.00062086 -2.4694 0.0135333 *
oc    -0.00699637  0.00155408 -4.5019 6.734e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log-Likelihood: -1008.2
McFadden R^2:  0.013691
Likelihood ratio test : chisq = 27.99 (p.value=8.3572e-07)

R> apply(fitted(mc, outcome = FALSE), 2, mean)

        hp         ec         er         gc         gr
0.05555556 0.07111111 0.09333333 0.63666667 0.14333333
```

Note that they match exactly: alternative-specific constants in a logit model insure that the average probabilities equal the observed shares.

(b) Calculate the wtp and discount rate r that is implied by the estimates. Are these reasonable?

```
R> wtp <- coef(mc)["oc"]/coef(mc)["ic"]
R> wtp

      oc
4.563385

R> r <- 1/wtp
R> r

       oc
0.2191356
```

The decision-maker is willing to pay $4.56 for a $1 year stream of savings. This implies $r = 0.22$. The decision-maker applies a 22% discount rate. These results are certainly more reasonable that in the previous model. The decision-maker is still estimated to be valuing saving somewhat less than would seem rational (ie applying a higher discount rate than seems reasonable). However, we need to remember that the decision-maker here is the builder. If home buyers were perfectly informed, then the builder would adopt the buyer's discount rate. However, the builder would adopt a higher discount rate if home buyers were not perfectly informed about (or believed) the stream of saving.

(c) This model contains constants for all alternatives ec-er-gc-gr, with the constant for alternative hp normalized to zero. Suppose you had included constants for alternatives ec-er-gc-hp, with the constant for alternative gr normalized to zero. What would be the estimated coefficient of the constant for alternative gc? Figure this out logically rather than actually estimating the model.

We know that when the hp is left out, the constant for alternative gc is 1.71074 meaning that the average impact of unicluded factors is 1.71074 higher for alternative gc than for alternative hp. Similarly, the constant for alternative gr is 0.30777. If gr were left out instead of hp, then all the constants would be relative to alternative gr. The constant for alternative gc would the be $1.71074 - .30777 = 1.40297$. That is, the average impact of unincluded factors is 1.40297 higher for alt gc than alt gr. Similarly for the other alternatives. Note the the constant for alt 5 would be $0 - .30777 = -.3077$, since hp is normalized to zero in the model with hp left out.

```
R> update(mc, reflevel = "gr")

Call:
mlogit(formula = depvar ~ ic + oc, data = H, reflevel = "gr",     method = "nr", print.level =

Coefficients:
     altec       alter       altgc       althp          ic          oc
 1.3505827   1.5451737   1.4027160  -0.3082633  -0.0015332  -0.0069964
```

5. Now try some models with sociodemographic variables entering.

(a) Enter installation cost divided by income, instead of installation cost. With this specification, the magnitude of the installation cost coefficient is inversely related to income, such that high income households are less concerned with installation costs than lower income households. Does dividing installation cost by income seem to make the model better or worse?

```
R> mi <- mlogit(depvar ~ oc + I(ic/income), H)
R> summary(mi)

Call:
mlogit(formula = depvar ~ oc + I(ic/income), data = H, method = "nr",
    print.level = 0)

Frequencies of alternatives:
      ec       er       gc       gr       hp
0.071111 0.093333 0.636667 0.143333 0.055556

nr method
6 iterations, 0h:0m:0s
g'(-H)^-1g = 1.03E-05
successive fonction values within tolerance limits

Coefficients :
               Estimate Std. Error t-value  Pr(>|t|)
alter         0.0639934  0.1944893  0.3290  0.742131
altgc         0.0563481  0.4650251  0.1212  0.903555
altgr        -1.4653063  0.5033845 -2.9109  0.003604 **
althp        -1.8700773  0.4364248 -4.2850 1.827e-05 ***
oc           -0.0071066  0.0015518 -4.5797 4.657e-06 ***
I(ic/income) -0.0027658  0.0018944 -1.4600  0.144298
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log-Likelihood: -1010.2
McFadden R^2:  0.011765
Likelihood ratio test : chisq = 24.052 (p.value=5.9854e-06)
```

The model seems to get worse. The LL is lower (more negative) and the coefficient on installation cost becomes insignificant (t-stat below 2).

(b) Instead of dividing installation cost by income, enter alternative-specific income effects. What do the estimates imply about the impact of income on the choice of central systems versus room system? Do these income terms enter significantly?

```
R> mi2 <- mlogit(depvar ~ oc + ic | income, H, reflevel = "hp")
```

The model implies that as income rises, the probability of heat pump rises relative to all the others (since income in the heat pump alt is normalized to zero, and the others enter with negative signs such that they are lower than that for heat pumps. Also, as income rises, the probability of gas room drops relative to the other non-heat-pump systems (since it is most negative).

Do these income terms enter significantly? No. It seems that income doesn't really have an effect. Maybe this is because income is for the family that lives in the house, whereas the builder made decision of which system to install.

```
R> lrtest(mc, mi2)

Likelihood ratio test

Model 1: depvar ~ ic + oc
Model 2: depvar ~ oc + ic | income
  #Df  LogLik Df  Chisq Pr(>Chisq)
1   6 -1008.2
2  10 -1005.9  4 4.6803     0.3217

R> waldtest(mc, mi2)

Wald test

Model 1: depvar ~ ic + oc
Model 2: depvar ~ oc + ic | income
  Res.Df Df  Chisq Pr(>Chisq)
1    894
2    890  4 4.6456     0.3256

R> scoretest(mc, mi2)

        score test

data:  depvar ~ oc + ic | income
chisq = 4.6761, = 4, p-value = 0.3222
alternative hypothesis: unconstrained model
```

(c) Try other models. Determine which model you think is best from these data.

I'm not going to give what I consider my best model: your ideas on what's best are what matter here.

6. We now are going to consider the use of logit model for prediction. Estimate a model with installation costs, operating costs, and alternative specific constants. Calculate the probabilities for each house explicitly. Check to be sure that the mean probabilities are the same as you got in exercise 4.

```
R> X <- model.matrix(mc)
R> alt <- index(H)$alt
R> chid <- index(H)$chid
R> eXb <- as.numeric(exp(X %*% coef(mc)))
R> SeXb <- tapply(eXb, chid, sum)
R> P <- eXb/SeXb[chid]
R> P <- matrix(P, ncol = 5, byrow = TRUE)
R> head(P)
```

```
           [,1]       [,2]      [,3]      [,4]       [,5]
[1,] 0.05107444 0.07035738 0.6329116 0.1877416 0.05791494
[2,] 0.04849337 0.06420595 0.6644519 0.1558322 0.06701658
[3,] 0.07440281 0.08716904 0.6387765 0.1439919 0.05565974
[4,] 0.07264503 0.11879833 0.5657376 0.1879231 0.05489595
[5,] 0.09223575 0.10238514 0.5670663 0.1561227 0.08219005
[6,] 0.09228184 0.10466584 0.6366615 0.1152634 0.05112739
```

```
R> apply(P, 2, mean)
```

```
[1] 0.07111111 0.09333333 0.63666666 0.14333334 0.05555556
```

7. The California Energy Commission (CEC) is considering whether to offer rebates on heat pumps. The CEC wants to predict the effect of the rebates on the heating system choices of customers in California. The rebates will be set at 10% of the installation cost. Using the estimated coefficients from the model in exercise 6, calculate new probabilities and predicted shares using the new installation cost of heat pump. How much do the rebates raise the share of houses with heat pumps?

```
R> X <- model.matrix(mc)
R> X[alt == "hp", "ic"] <- X[alt == "hp", "ic"] * 0.9
R> eXb <- as.numeric(exp(X %*% coef(mc)))
R> SeXb <- tapply(eXb, chid, sum)
R> P <- eXb/SeXb[chid]
R> P <- matrix(P, ncol = 5, byrow = TRUE)
R> apply(P, 2, mean)
```

```
[1] 0.07045486 0.09247026 0.63064443 0.14196814 0.06446230
```

We estimate the model with the actual costs. Then we change the costs and calculate probabilities with the new costs. The average probability is the predicted share for an alternative. At the original costs, the heat pump share is 0.0555 (ie, about 5.5%) This share is predicted to rise to 0.0645 (about 6.5%) when rebates are given.

8. Suppose a new technology is developed that provides more efficient central heating. The new technology costs \$200 more than the central electric system. However, it saves 25% of the electricity, such that its operating costs are 75% of the operating costs of ec. We want to predict the potential market penetration of this technology. Note that there are now six alternatives: the original five alternatives plus this new one. Calculate the probability and predict the market share (i.e., the average probability) for all six alternatives, using the model that is estimated on the original five alternatives. (Be sure to use the original installation cost for heat pumps, rather than the reduced cost in exercise 7.) What is the predicted market share for the new technology? From which of the original five systems does the new technology draw the most customers?

```
R> X <- model.matrix(mc)
R> Xn <- X[alt == "ec", ]
R> Xn[, "ic"] <- Xn[, "ic"] + 200
R> Xn[, "oc"] <- Xn[, "oc"] * 0.75
R> unchid <- unique(index(H)$chid)
```

```
R> rownames(Xn) <- paste(unchid, "new", sep = ".")
R> chidb <- c(chid, unchid)
R> X <- rbind(X, Xn)
R> X <- X[order(chidb), ]
R> eXb <- as.numeric(exp(X %*% coef(mc)))
R> SeXb <- as.numeric(tapply(eXb, sort(chidb), sum))
R> P <- eXb/SeXb[sort(chidb)]
R> P <- matrix(P, ncol = 6, byrow = TRUE)
R> apply(P, 2, mean)
```

```
[1] 0.06311578 0.08347713 0.57145108 0.12855080 0.04977350 0.10363170
```

The new technology captures a market share of 0.1036. That is, it gets slightly more than ten percent of the market.

It draws the same percent (about 10%) from each system. This means that it draws the most in absolute terms from the most popular system, gas central. For example, gas central drops from to 0.637 to 0.571; this is an absolute drop of $0.637 - 0.571 = 0.065$ and a percent drop of $0.065/0.637$ about 10%. Of the 10.36% market share that is attained by the new technology, 6.5% of it comes from gas central. The other systems drop by about the same percent, which is less in absolute terms.

The same percent drop for all systems is a consequence of the IIA property of logit. To me, this property seems unreasonable in this application. The new technology is a type of electric system. It seems reasonable that it would draw more from other electric systems than from gas systems. Models like nested logit, probit, and mixed logit allow more flexible, and in this case, more realistic substitution patterns.

2. Nested logit model

The data set HC from **mlogit** contains data in R format on the choice of heating and central cooling system for 250 single-family, newly built houses in California.

The alternatives are:

1. Gas central heat with cooling (gcc)
2. Electric central resistence heat with cooling (ecc)
3. Electric room resistence heat with cooling (erc)
4. Electric heat pump, which provides cooling also (hpc)
5. Gas central heat without cooling (gc)
6. Electric central resistence heat without cooling (ec)
7. Electric room resistence heat without cooling (er)

Heat pumps necessarily provide both heating and cooling such that heat pump without cooling is not an alternative.

The variables are:

- depvar gives the name of the chosen alternative,
- ich.alt are the installation cost for the heating portion of the system,
- icca is the installation cost for cooling
- och.alt are the operating cost for the heating portion of the system
- occa is the operating cost for cooling
- income is the annual income of the household

Note that the full installation cost of alternative gcc is ich.gcc+icca, and similarly for the operating cost and for the other alternatives with cooling.

1. Run a nested logit model on the data for two nests and one log-sum coefficient that applies to both nests. Note that the model is specified to have the cooling alternatives (gcc, ecc, erc, hpc) in one nest and the non-cooling alternatives (gc, ec, er) in another nest.

```
R> library("mlogit")
R> data("HC", package = "mlogit")
R> HC <- mlogit.data(HC, varying = c(2:8, 10:16), choice = "depvar",
+     shape = "wide")
R> cooling.modes <- index(HC)$alt %in% c("gcc", "ecc", "erc", "hpc")
R> room.modes <- index(HC)$alt %in% c("erc", "er")
R> HC$icca[!cooling.modes] <- 0
R> HC$occa[!cooling.modes] <- 0
R> HC$inc.cooling <- HC$inc.room <- 0
R> HC$inc.cooling[cooling.modes] <- HC$income[cooling.modes]
R> HC$inc.room[room.modes] <- HC$income[room.modes]
R> HC$int.cooling <- as.numeric(cooling.modes)
R> nl <- mlogit(depvar ~ ich + och + icca + occa + inc.room + inc.cooling +
+     int.cooling | 0, HC, nests = list(cooling = c("gcc", "ecc",
+     "erc", "hpc"), other = c("gc", "ec", "er")), un.nest.el = TRUE)
R> summary(nl)
```

```
Call:
mlogit(formula = depvar ~ ich + och + icca + occa + inc.room +
    inc.cooling + int.cooling | 0, data = HC, nests = list(cooling = c("gcc",
    "ecc", "erc", "hpc"), other = c("gc", "ec", "er")), un.nest.el = TRUE)

Frequencies of alternatives:
   ec   ecc    er   erc    gc   gcc   hpc
0.004 0.016 0.032 0.004 0.096 0.744 0.104

bfgs method
11 iterations, 0h:0m:1s
g'(-H)^-1g = 7.26E-06
successive fonction values within tolerance limits
```

```
Coefficients :
             Estimate Std. Error t-value  Pr(>|t|)
ich         -0.554878   0.144205 -3.8478 0.0001192 ***
och         -0.857886   0.255313 -3.3601 0.0007791 ***
icca        -0.225079   0.144423 -1.5585 0.1191212
occa        -1.089458   1.219821 -0.8931 0.3717882
inc.room    -0.378971   0.099631 -3.8038 0.0001425 ***
inc.cooling  0.249575   0.059213  4.2149 2.499e-05 ***
int.cooling -6.000415   5.562423 -1.0787 0.2807030
iv           0.585922   0.179708  3.2604 0.0011125 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log-Likelihood: -178.12
```

(a) The estimated log-sum coefficient is -0.59. What does this estimate tell you about the degree of correlation in unobserved factors over alternatives within each nest?

The correlation is approximately $1 - 0.59 = 0.41$. It's a moderate correlation.

(b) Test the hypothesis that the log-sum coefficient is 1.0 (the value that it takes for a standard logit model.) Can the hypothesis that the true model is standard logit be rejected?

We can use a t-test of the hypothesis that the log-sum coefficient equal to 1. The t-statistic is :

```
R> (coef(nl)["iv"] - 1)/sqrt(vcov(nl)["iv", "iv"])

       iv
-2.304171
```

The critical value of t for 95% confidence is 1.96. So we can reject the hypothesis at 95% confidence.

We can also use a likelihood ratio test because the multinomial logit is a special case of the nested model.

```
R> ml <- update(nl, nests = NULL)
R> lrtest(nl, ml)

Likelihood ratio test

Model 1: depvar ~ ich + och + icca + occa + inc.room + inc.cooling + int.cooling |
    0
Model 2: depvar ~ ich + och + icca + occa + inc.room + inc.cooling + int.cooling |
    0
  #Df  LogLik Df  Chisq Pr(>Chisq)
1   8 -178.12
2   7 -180.29 -1 4.3234    0.03759 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Note that the hypothesis is rejected at 95% confidence, but not at 99% confidence.

2. Re-estimate the model with the room alternatives in one nest and the central alternatives in another nest. (Note that a heat pump is a central system.)

```
R> nl2 <- update(nl, nests = list(central = c("ec", "ecc", "gc",
+     "gcc", "hpc"), room = c("er", "erc")))
R> summary(nl2)

Call:
mlogit(formula = depvar ~ ich + och + icca + occa + inc.room +
    inc.cooling + int.cooling | 0, data = HC, nests = list(central = c("ec",
    "ecc", "gc", "gcc", "hpc"), room = c("er", "erc")), un.nest.el = TRUE)

Frequencies of alternatives:
   ec   ecc    er   erc    gc   gcc   hpc
0.004 0.016 0.032 0.004 0.096 0.744 0.104

bfgs method
10 iterations, 0h:0m:1s
g'(-H)^-1g = 5.87E-07
gradient close to zero

Coefficients :
             Estimate Std. Error t-value Pr(>|t|)
ich          -1.13818    0.54216 -2.0993  0.03579 *
och          -1.82532    0.93228 -1.9579  0.05024 .
icca         -0.33746    0.26934 -1.2529  0.21024
occa         -2.06328    1.89726 -1.0875  0.27681
inc.room     -0.75722    0.34292 -2.2081  0.02723 *
inc.cooling   0.41689    0.20742  2.0099  0.04444 *
int.cooling -13.82487    7.94031 -1.7411  0.08167 .
iv            1.36201    0.65393  2.0828  0.03727 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log-Likelihood: -180.02
```

(a) What does the estimate imply about the substitution patterns across alternatives? Do you think the estimate is plausible?

The log-sum coefficient is over 1. This implies that there is more substitution across nests than within nests. I don't think this is very reasonable, but people can differ on their concepts of what's reasonable.

(b) Is the log-sum coefficient significantly different from 1?

The t-statistic is :

```
R> (coef(nl2)["iv"] - 1)/sqrt(vcov(nl2)["iv", "iv"])

       iv
0.5535849

R> lrtest(nl2, ml)

Likelihood ratio test

Model 1: depvar ~ ich + och + icca + occa + inc.room + inc.cooling + int.cooling |
    0
Model 2: depvar ~ ich + och + icca + occa + inc.room + inc.cooling + int.cooling |
    0
  #Df  LogLik Df  Chisq Pr(>Chisq)
1   8 -180.02
2   7 -180.29 -1 0.5268      0.468
```

We cannot reject the hypothesis at standard confidence levels.

(c) How does the value of the log-likelihood function compare for this model relative to the model in exercise 1, where the cooling alternatives are in one nest and the heating alternatives in the other nest.

```
R> logLik(nl)

'log Lik.' -178.1247 (df=8)

R> logLik(nl2)

'log Lik.' -180.0231 (df=8)
```

The LL is worse (more negative.) All in all, this seems like a less appropriate nesting structure.

3. Rerun the model that has the cooling alternatives in one nest and the non-cooling alternatives in the other nest (like for exercise 1), with a separate log-sum coefficient for each nest.

```
R> nl3 <- update(nl, un.nest.el = FALSE)
```

(a) Which nest is estimated to have the higher correlation in unobserved factors? Can you think of a real-world reason for this nest to have a higher correlation?

The correlation in the cooling nest is around 1-0.60 = 0.4 and that for the non-cooling nest is around 1-0.45 = 0.55. So the correlation is higher in the non-cooling nest. Perhaps more variation in comfort when there is no cooling. This variation in comfort is the same for all the non-cooling alternatives.

(b) Are the two log-sum coefficients significantly different from each other? That is, can you reject the hypothesis that the model in exercise 1 is the true model?

We can use a likelihood ratio tests with models `nl` and `nl3`.

```
R> lrtest(nl, nl3)

Likelihood ratio test

Model 1: depvar ~ ich + och + icca + occa + inc.room + inc.cooling + int.cooling |
    0
Model 2: depvar ~ ich + och + icca + occa + inc.room + inc.cooling + int.cooling |
    0
  #Df  LogLik Df  Chisq Pr(>Chisq)
1   8 -178.12
2   9 -178.04  1 0.1758      0.675
```

The restricted model is the one from exercise 1 that has one log-sum coefficient. The unrestricted model is the one we just estimated. The test statistics is 0.6299. The critical value of chi-squared with 1 degree of freedom is 3.8 at the 95% confidence level. We therefore cannot reject the hypothesis that the two nests have the same log-sum coefficient.

4. Rewrite the code to allow three nests. For simplicity, estimate only one log-sum coefficient which is applied to all three nests. Estimate a model with alternatives gcc, ecc and erc in a nest, hpc in a nest alone, and alternatives gc, ec and er in a nest. Does this model seem better or worse than the model in exercise 1, which puts alternative hpc in the same nest as alternatives gcc, ecc and erc?

```
R> nl4 <- update(nl, nests = list(n1 = c("gcc", "ecc", "erc"), n2 = c("hpc"),
+     n3 = c("gc", "ec", "er")))
R> summary(nl4)

Call:
mlogit(formula = depvar ~ ich + och + icca + occa + inc.room +
    inc.cooling + int.cooling | 0, data = HC, nests = list(n1 = c("gcc",
    "ecc", "erc"), n2 = c("hpc"), n3 = c("gc", "ec", "er")),
    un.nest.el = TRUE)

Frequencies of alternatives:
   ec   ecc    er   erc    gc   gcc   hpc
0.004 0.016 0.032 0.004 0.096 0.744 0.104

bfgs method
8 iterations, 0h:0m:1s
g'(-H)^-1g = 3.71E-08
gradient close to zero

Coefficients :
             Estimate Std. Error t-value  Pr(>|t|)
ich         -0.838394   0.100546 -8.3384 < 2.2e-16 ***
och         -1.331598   0.252069 -5.2827 1.273e-07 ***
icca        -0.256131   0.145564 -1.7596   0.07848 .
occa        -1.405656   1.207281 -1.1643   0.24430
inc.room    -0.571352   0.077950 -7.3297 2.307e-13 ***
inc.cooling  0.311355   0.056357  5.5247 3.301e-08 ***
```

```
int.cooling -10.413384   5.612445 -1.8554   0.06354 .
iv            0.956544   0.180722  5.2929 1.204e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log-Likelihood: -180.26
```

The LL for this model is -180.26, which is lower (more negative) than for the model with two nests, which got -178.12.

3. Mixed logit model

A sample of residential electricity customers were asked a series of choice experiments. In each experiment, four hypothetical electricity suppliers were described. The person was asked which of the four suppliers he/she would choose. As many as 12 experiments were presented to each person. Some people stopped before answering all 12. There are 361 people in the sample, and a total of 4308 experiments. In the experiments, the characteristics of each supplier were stated. The price of the supplier was either :

1. a fixed price at a stated cents per kWh, with the price varying over suppliers and experiments.

2. a time-of-day (TOD) rate under which the price is 11 cents per kWh from 8am to 8pm and 5 cents per kWh from 8pm to 8am. These TOD prices did not vary over suppliers or experiments: whenever the supplier was said to offer TOD, the prices were stated as above.

3. a seasonal rate under which the price is 10 cents per kWh in the summer, 8 cents per kWh in the winter, and 6 cents per kWh in the spring and fall. Like TOD rates, these prices did not vary. Note that the price is for the electricity only, not transmission and distribution, which is supplied by the local regulated utility.

The length of contract that the supplier offered was also stated, in years (such as 1 year or 5 years.) During this contract period, the supplier guaranteed the prices and the buyer would have to pay a penalty if he/she switched to another supplier. The supplier could offer no contract in which case either side could stop the agreement at any time. This is recorded as a contract length of 0.

Some suppliers were also described as being a local company or a "well-known" company. If the supplier was not local or well-known, then nothing was said about them in this regard [1].

1. Run a mixed logit model without intercepts and a normal distribution for the 6 parameters of the model, using 100 draws, halton sequences and taking into account the panel data structure.

[1] These are the data used in Revelt and Train (2000) and Hubert and Train (2001).

```
R> library("mlogit")
R> data("Electricity", package = "mlogit")
R> Electr <- mlogit.data(Electricity, id = "id", choice = "choice",
+     varying = 3:26, shape = "wide", sep = "")

R> Elec.mxl <- mlogit(choice ~ pf + cl + loc + wk + tod + seas |
+     0, Electr, rpar = c(pf = "n", cl = "n", loc = "n", wk = "n",
+     tod = "n", seas = "n"), R = 100, halton = NA, print.level = 0,
+     panel = TRUE)

R> summary(Elec.mxl)

Call:
mlogit(formula = choice ~ pf + cl + loc + wk + tod + seas | 0,
    data = Electr, rpar = c(pf = "n", cl = "n", loc = "n", wk = "n",
        tod = "n", seas = "n"), R = 100, halton = NA, panel = TRUE,
    print.level = 0)

Frequencies of alternatives:
      1       2       3       4
0.22702 0.26393 0.23816 0.27089

bfgs method
24 iterations, 0h:1m:21s
g'(-H)^-1g = 8.04E-07
gradient close to zero

Coefficients :
         Estimate Std. Error t-value  Pr(>|t|)
pf      -0.965116   0.033962 -28.417 < 2.2e-16 ***
cl      -0.202566   0.013161 -15.392 < 2.2e-16 ***
loc      2.090584   0.081615  25.615 < 2.2e-16 ***
wk       1.486831   0.065932  22.551 < 2.2e-16 ***
tod     -9.135941   0.288321 -31.687 < 2.2e-16 ***
seas    -9.348937   0.290597 -32.172 < 2.2e-16 ***
sd.pf    0.196812   0.010491  18.761 < 2.2e-16 ***
sd.cl    0.352922   0.017315  20.382 < 2.2e-16 ***
sd.loc   1.686309   0.091258  18.479 < 2.2e-16 ***
sd.wk    1.148274   0.079385  14.465 < 2.2e-16 ***
sd.tod   2.479706   0.129110  19.206 < 2.2e-16 ***
sd.seas  1.190922   0.100000  11.909 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Log-Likelihood: -3947.2

random coefficients
     Min.     1st Qu.     Median       Mean     3rd Qu. Max.
pf   -Inf  -1.0978644 -0.9651164 -0.9651164 -0.83236847  Inf
cl   -Inf  -0.4406087 -0.2025662 -0.2025662  0.03547624  Inf
loc  -Inf   0.9531858  2.0905840  2.0905840  3.22798226  Inf
wk   -Inf   0.7123318  1.4868311  1.4868311  2.26133031  Inf
tod  -Inf -10.8084767 -9.1359407 -9.1359407 -7.46340468  Inf
seas -Inf -10.1522014 -9.3489366 -9.3489366 -8.54567177  Inf
```

2. (a) Using the estimated mean coefficients, determine the amount that a customer with average coefficients for price and length is willing to pay for an extra year of contract length.

```
R> coef(Elec.mxl)["cl"]/coef(Elec.mxl)["pf"]

       cl
0.2098879
```

The mean coefficient of length is -0.20. The consumer with this average coefficient dislikes having a longer contract. So this person is willing to pay to reduce the length of the contract. The mean price coefficient is -0.97. A customer with these coefficients is willing to pay 0.20/0.97=0.21, or one-fifth a cent per kWh extra to have a contract that is one year shorter.

(b) Determine the share of the population who are estimated to dislike long term contracts (ie have a negative coefficient for the length.)

```
R> pnorm(-coef(Elec.mxl)["cl"]/coef(Elec.mxl)["sd.cl"])

       cl
0.7170054
```

The coefficient of length is normally distributed with mean -0.20 and standard deviation 0.35. The share of people with coefficients below zero is the cumulative probability of a standardized normal deviate evaluated at 0.20/0.35=0.57. Looking 0.57 up in a table of the standard normal distribution, we find that the share below 0.57 is 0.72. About seventy percent of the population are estimated to dislike long-term contracts.

3. The price coefficient is assumed to be normally distributed in these runs. This assumption means that some people are assumed to have positive price coefficients, since the normal distribution has support on both sides of zero. Using your estimates from exercise 1, determine the share of customers with positive price coefficients. As you can see, this is pretty small share and can probably be ignored. However, in some situations, a normal distribution for the price coefficient will give a fairly large share with the wrong sign. Revise the program to make the price coefficient fixed rather than random. A fixed price coefficient also makes it easier to calculate the distribution of willingness to pay (wtp) for each non-price attribute. If the price coefficient is fixed, the

distribtion of wtp for an attribute has the same distribution as the attribute's coefficient, simply scaled by the price coefficient. However, when the price coefficient is random, the distribution of wtp is the ratio of two distributions, which is harder to work with.

```
R> pnorm(-coef(Elec.mxl)["pf"]/coef(Elec.mxl)["sd.pf"])
```

```
       pf
0.9999995
```

The price coefficient is distributed normal with mean -0.97 and standard deviation 0.20. The cumulative standard normal distribution evaluated at 0.97/0.20=4.85 is more than 0.999, which means that more than 99.9% of the population are estimated to have negative price coefficients. Essentially no one is estimated to have a positive price coefficient.

```
R> Elec.mxl2 <- mlogit(choice ~ pf + cl + loc + wk + tod + seas |
+     0, Electr, rpar = c(cl = "n", loc = "n", wk = "n", tod = "n",
+     seas = "n"), R = 100, halton = NA, print.level = 0, panel = TRUE)
```

```
R> summary(Elec.mxl2)
```

```
Call:
mlogit(formula = choice ~ pf + cl + loc + wk + tod + seas | 0,
    data = Electr, rpar = c(cl = "n", loc = "n", wk = "n", tod = "n",
        seas = "n"), R = 100, halton = NA, panel = TRUE, print.level = 0)

Frequencies of alternatives:
      1       2       3       4
0.22702 0.26393 0.23816 0.27089

bfgs method
22 iterations, 0h:1m:17s
g'(-H)^-1g = 8.15E-07
gradient close to zero

Coefficients :
         Estimate Std. Error t-value  Pr(>|t|)
pf      -0.880752   0.032642 -26.982 < 2.2e-16 ***
cl      -0.211077   0.013337 -15.827 < 2.2e-16 ***
loc      2.128798   0.079884  26.649 < 2.2e-16 ***
wk       1.531639   0.065426  23.410 < 2.2e-16 ***
tod     -8.411831   0.278401 -30.215 < 2.2e-16 ***
seas    -8.559577   0.278334 -30.753 < 2.2e-16 ***
sd.cl    0.400717   0.018981  21.111 < 2.2e-16 ***
sd.loc   1.390452   0.086054  16.158 < 2.2e-16 ***
sd.wk    0.944443   0.076032  12.422 < 2.2e-16 ***
sd.tod   2.940810   0.135510  21.702 < 2.2e-16 ***
sd.seas  2.025004   0.108474  18.668 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Log-Likelihood: -3979.1

random coefficients
     Min.     1st Qu.     Median       Mean     3rd Qu. Max.
cl   -Inf  -0.4813561 -0.2110767 -0.2110767  0.05920261  Inf
loc  -Inf   1.1909528  2.1287981  2.1287981  3.06664352  Inf
wk   -Inf   0.8946226  1.5316394  1.5316394  2.16865619  Inf
tod  -Inf -10.3953779 -8.4118315 -8.4118315 -6.42828507  Inf
seas -Inf  -9.9254211 -8.5595769 -8.5595769 -7.19373276  Inf
```

4. You think that everyone must like using a known company rather than an unknown one, and yet the normal distribution implies that some people dislike using a known company. Revise the program to give the coefficient of wk a uniform distribution (do this with the price coefficient fixed).

```
R> Elec.mxl3 <- update(Elec.mxl, rpar = c(cl = "n", loc = "n", wk = "u",
+     tod = "n", seas = "n"))
```

The price coefficient is uniformly distributed with parameters 1.541 and 1.585.

```
R> summary(Elec.mxl3)

Call:
mlogit(formula = choice ~ pf + cl + loc + wk + tod + seas | 0,
    data = Electr, rpar = c(cl = "n", loc = "n", wk = "u", tod = "n",
        seas = "n"), R = 100, halton = NA, panel = TRUE, print.level = 0)

Frequencies of alternatives:
      1       2       3       4
0.22702 0.26393 0.23816 0.27089

bfgs method
23 iterations, 0h:1m:23s
g'(-H)^-1g = 4.04E-07
gradient close to zero

Coefficients :
         Estimate Std. Error t-value  Pr(>|t|)
pf      -0.881536   0.032632 -27.015 < 2.2e-16 ***
cl      -0.214181   0.013432 -15.946 < 2.2e-16 ***
loc      2.117730   0.079765  26.550 < 2.2e-16 ***
wk       1.541424   0.065536  23.520 < 2.2e-16 ***
tod     -8.422855   0.277969 -30.301 < 2.2e-16 ***
seas    -8.536515   0.277423 -30.771 < 2.2e-16 ***
sd.cl    0.403138   0.018972  21.249 < 2.2e-16 ***
sd.loc   1.427939   0.086573  16.494 < 2.2e-16 ***
sd.wk    1.584958   0.122157  12.975 < 2.2e-16 ***
sd.tod   2.979155   0.136691  21.795 < 2.2e-16 ***
sd.seas  2.021477   0.107766  18.758 < 2.2e-16 ***
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log-Likelihood: -3976.5

random coefficients
           Min.     1st Qu.     Median       Mean     3rd Qu.     Max.
cl         -Inf  -0.4860938 -0.2141813 -0.2141813  0.05773125      Inf
loc        -Inf   1.1545994  2.1177296  2.1177296  3.08085982      Inf
wk   -0.04353385   0.7489451  1.5414240  1.5414240  2.33390299 3.126382
tod        -Inf -10.4322639 -8.4228546 -8.4228546 -6.41344518      Inf
seas       -Inf  -9.8999811 -8.5365154 -8.5365154 -7.17304978      Inf
```

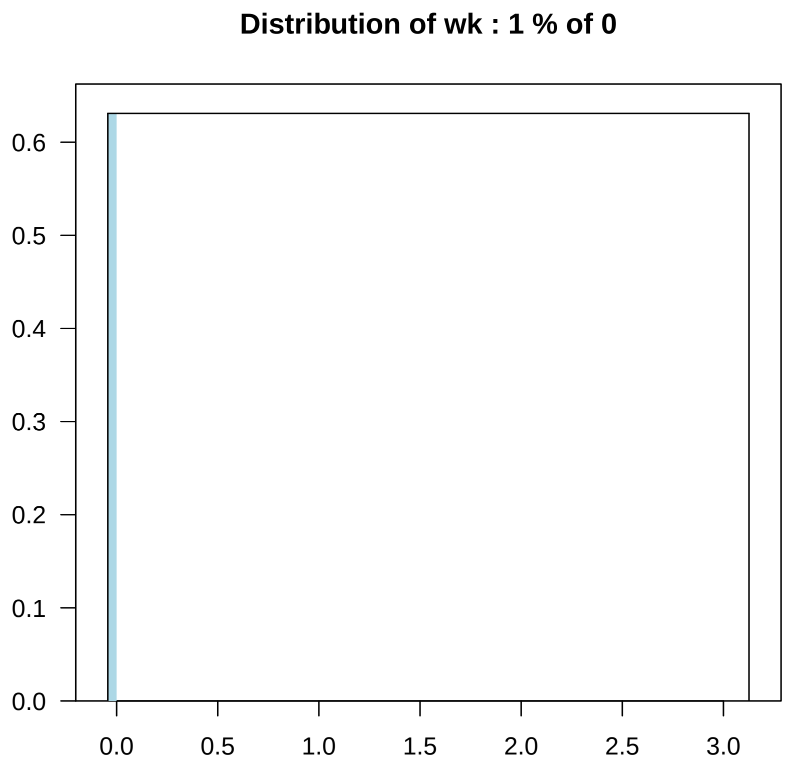
```
R> rpar(Elec.mxl3, "wk")
```

```
uniform distribution with parameters 1.541 (center) and 1.585 (span)
```

```
R> summary(rpar(Elec.mxl3, "wk"))
```

```
       Min.     1st Qu.      Median        Mean     3rd Qu.        Max.
-0.04353385  0.74894510  1.54142404  1.54142404  2.33390299  3.12638193
```

```
R> plot(rpar(Elec.mxl3, "wk"))
```



The upper bound is 3.13. The estimated price coefficient is -0.88 and so the willingness to pay for a known provided ranges uniformly from -0.05 to 3.55 cents per kWh.

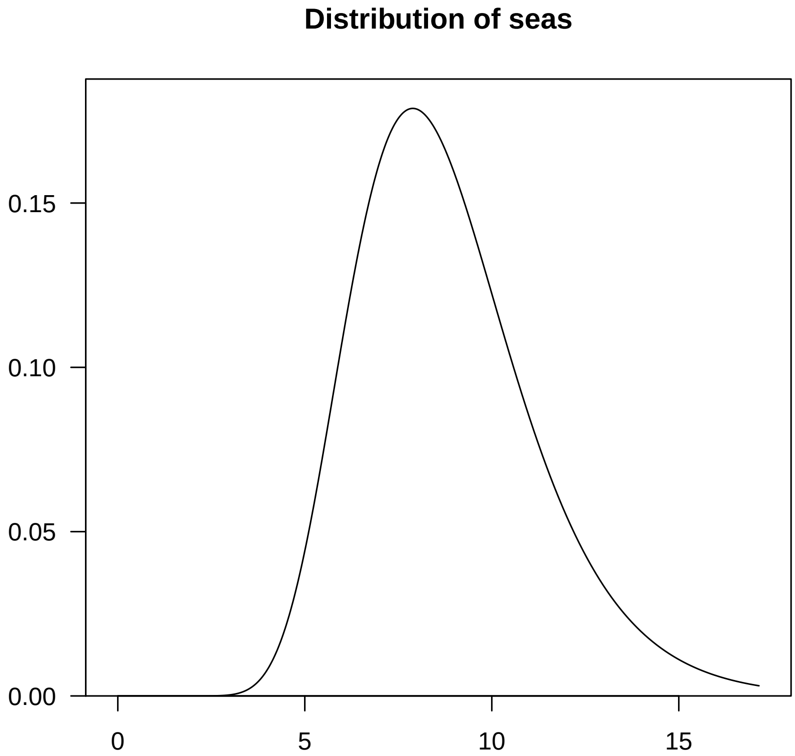
7. Rerun the model with a fixed coefficient for price and lognormal distributions for the coefficients of TOD and seasonal (since their coefficient should be negative for all people.) To do this, you need to reverse the sign of the TOD and seasonal variables, since the lognormal is always positive and you want the these coefficients to be always negative.

A lognormal is specified as $\exp(b + se)$ where e is a standard normal deviate. The parameters of the lognormal are b and s. The mean of the lognormal is $\exp(b + 0.5s^2)$ and the standard deviation is the mean times $\sqrt{(\exp(s^2)) - 1}$.

```
R> Electr <- mlogit.data(Electricity, id = "id", choice = "choice",
+     varying = 3:26, shape = "wide", sep = "", opposite = c("tod",
+         "seas"))
R> Elec.mxl4 <- mlogit(choice ~ pf + cl + loc + wk + tod + seas |
+     0, Electr, rpar = c(cl = "n", loc = "n", wk = "u", tod = "ln",
+     seas = "ln"), R = 100, halton = NA, print.level = 0, panel = TRUE)

R> summary(Elec.mxl4)
```

```
Call:
mlogit(formula = choice ~ pf + cl + loc + wk + tod + seas | 0,
    data = Electr, rpar = c(cl = "n", loc = "n", wk = "u", tod = "ln",
        seas = "ln"), R = 100, halton = NA, panel = TRUE, print.level = 0)

Frequencies of alternatives:
      1       2       3       4
0.22702 0.26393 0.23816 0.27089

bfgs method
20 iterations, 0h:1m:13s
g'(-H)^-1g = 7.94E-07
gradient close to zero

Coefficients :
         Estimate Std. Error t-value  Pr(>|t|)
pf      -0.864400   0.032280 -26.778 < 2.2e-16 ***
cl      -0.186722   0.012983 -14.382 < 2.2e-16 ***
loc      2.111908   0.080464  26.247 < 2.2e-16 ***
wk       1.468139   0.065146  22.536 < 2.2e-16 ***
tod      2.167303   0.032957  65.761 < 2.2e-16 ***
seas     2.139594   0.033242  64.364 < 2.2e-16 ***
sd.cl    0.364684   0.017842  20.439 < 2.2e-16 ***
sd.loc   1.651097   0.093066  17.741 < 2.2e-16 ***
sd.wk    1.544719   0.119748  12.900 < 2.2e-16 ***
sd.tod   0.400823   0.022656  17.692 < 2.2e-16 ***
sd.seas  0.272565   0.016727  16.295 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log-Likelihood: -3972.5

random coefficients
            Min.    1st Qu.     Median       Mean     3rd Qu.     Max.
cl          -Inf -0.4326980 -0.1867225 -0.1867225  0.05925309      Inf
loc         -Inf  0.9982596  2.1119076  2.1119076  3.22555558      Inf
wk   -0.07657984  0.6957796  1.4681390  1.4681390  2.24049839 3.012858
tod   0.00000000  6.6655463  8.7346934  9.4652987 11.44615399      Inf
seas  0.00000000  7.0692296  8.4959841  8.8175085 10.21069481      Inf
```

```
R> plot(rpar(Elec.mxl4, "seas"))
```



8. Rerun the same model as previously, but allowing now the correlation between random parameters. Compute the correlation matrix of the random parameters. Test the hypothesis that the random parameters are uncorrelated.

```
R> Elec.mxl5 <- update(Elec.mxl4, correlation = TRUE)

R> summary(Elec.mxl5)

Call:
mlogit(formula = choice ~ pf + cl + loc + wk + tod + seas | 0,
    data = Electr, rpar = c(cl = "n", loc = "n", wk = "u", tod = "ln",
        seas = "ln"), R = 100, correlation = TRUE, halton = NA,
    panel = TRUE, print.level = 0)

Frequencies of alternatives:
      1       2       3       4
0.22702 0.26393 0.23816 0.27089

bfgs method
29 iterations, 0h:1m:49s
g'(-H)^-1g = 6.78E-07
```

```
gradient close to zero

Coefficients :
           Estimate Std. Error  t-value  Pr(>|t|)
pf       -0.9087879  0.0341841 -26.5851 < 2.2e-16 ***
cl       -0.2146479  0.0137617 -15.5974 < 2.2e-16 ***
loc       2.4914154  0.0900551  27.6655 < 2.2e-16 ***
wk        1.8622769  0.0751320  24.7867 < 2.2e-16 ***
tod       2.1547504  0.0342944  62.8310 < 2.2e-16 ***
seas      2.1830322  0.0334489  65.2648 < 2.2e-16 ***
cl.cl     0.3446144  0.0171958  20.0406 < 2.2e-16 ***
cl.loc    0.3691683  0.0833319   4.4301 9.419e-06 ***
cl.wk     0.0905285  0.0678906   1.3334    0.1824
cl.tod   -0.0118224  0.0098980  -1.1944    0.2323
cl.seas   0.0510357  0.0090869   5.6164 1.950e-08 ***
loc.loc  -2.3298035  0.1205923 -19.3197 < 2.2e-16 ***
loc.wk   -1.5265142  0.0991530 -15.3955 < 2.2e-16 ***
loc.tod   0.1159167  0.0145357   7.9746 1.554e-15 ***
loc.seas  0.0533834  0.0115350   4.6279 3.693e-06 ***
wk.wk     0.8798520  0.0739828  11.8927 < 2.2e-16 ***
wk.tod   -0.0035827  0.0114302  -0.3134    0.7539
wk.seas   0.0069436  0.0109054   0.6367    0.5243
tod.tod   0.4162663  0.0229150  18.1657 < 2.2e-16 ***
tod.seas  0.1306154  0.0121220  10.7751 < 2.2e-16 ***
seas.seas 0.2165173  0.0136024  15.9176 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log-Likelihood: -3815.8

random coefficients
           Min.    1st Qu.     Median       Mean     3rd Qu.     Max.
cl         -Inf -0.4470868 -0.2146479 -0.2146479  0.01779100      Inf
loc        -Inf  0.9003816  2.4914154  2.4914154  4.08244930      Inf
wk   0.09802622  0.9801516  1.8622769  1.8622769  2.74440228 3.626528
tod  0.00000000  6.4442046  8.6257366  9.4705201 11.54577436      Inf
seas 0.00000000  7.4282527  8.8731705  9.1866703 10.59914853      Inf
```

```
R> cor.mlogit(Elec.mxl5)
```

```
              cl        loc          wk         tod       seas
cl    1.00000000  0.1565021  0.05131271 -0.02734892  0.1936691
loc   0.15650214  1.0000000  0.86261653 -0.26912704 -0.1697723
wk    0.05131271  0.8626165  1.00000000 -0.23755387 -0.1522019
tod  -0.02734892 -0.2691270 -0.23755387  1.00000000  0.5261005
seas  0.19366913 -0.1697723 -0.15220185  0.52610050  1.0000000
```

```
R> lrtest(Elec.mxl5, Elec.mxl4)
```

```
Likelihood ratio test
```

```
Model 1: choice ~ pf + cl + loc + wk + tod + seas | 0
Model 2: choice ~ pf + cl + loc + wk + tod + seas | 0
  #Df  LogLik  Df  Chisq Pr(>Chisq)
1  21 -3815.8
2  11 -3972.5 -10 313.26  < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

R> waldtest(Elec.mxl5, correlation = FALSE)

        Wald test

data:  uncorrelated random effects
chisq = 417.9007, df = 10, p-value < 2.2e-16

R> scoretest(Elec.mxl4, correlation = TRUE)

        score test

data:  correlation = TRUE
chisq = 2795.714, df = 10, p-value < 2.2e-16
alternative hypothesis: uncorrelated random effects
```

The three tests clearly reject the hypothesis that the random parameters are uncorrelated.

References

Hubert J, Train K (2001). "On the similarity of classical and Bayesian estimates of individual mean pathworths." *Marketing Letters*, **12**, 259–269.

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Contents

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